Geometry as a Semiclassical Effect in a Quantum World - Emergent gravity from matrix models

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Motivation

- incompatibility between GR and QFT

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \langle T_{\mu\nu}\rangle$$

lhs: classical Einstein tensor, rhs: ev of an operator

- natural limit in experimental length resolution: better length resolution requires higher energy, energy required for resolution of the Planck length has a Schwarzschild radius of the Planck length

$$\Delta x_\mu \simeq \lambda_p = \sqrt{\frac{G\hbar}{c^3}} \simeq 10^{-33}\mathrm{cm}$$

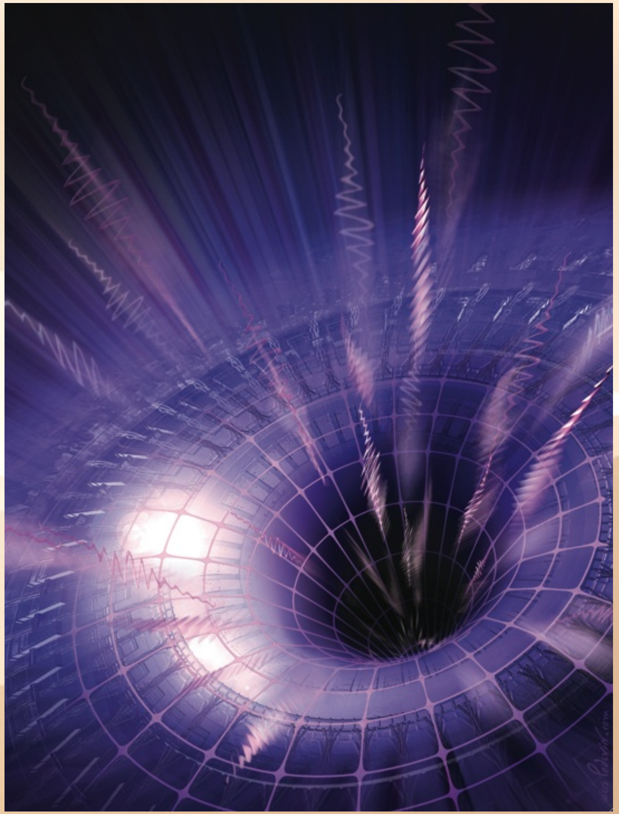


Image source:
http://web.physics.ucsb.edu/~giddings/sbgw/physics.html

How to define a "non-commutative" space?

Geometric space	⟷	commutative C*-algebra (according to Gel'fand-Naimark theorem)
Non-commutative space	⟷	Non-commutative C*-algebra (A. Connes 1994)

Weyl quantization

assume non-commuting space-time coordinates:

$[\hat{x}^\mu, \hat{x}^\nu] = i\theta^{\mu\nu}\,, \quad \Rightarrow$ leads to uncertainty relation $\quad \Delta x^\mu \Delta x^\nu \geq \frac{1}{2}|\theta^{\mu\nu}| \sim (\lambda_p)^2$

exists isomorphism mapping between NC algebra and commutative one, e.g. Weyl map

$$W : \mathcal{A} \to \widehat{\mathcal{A}}\,, \qquad x^i \mapsto \hat{x}^i$$

introduce a functions Weyl operator by

$\hat{\mathcal{W}}[f] := \int d^D x\, f(x)\hat{\Delta}(x)\,,$	$\hat{\Delta}(x) = \int \frac{d^D k}{(2\pi)^D} e^{ik_\mu \hat{x}^\mu} e^{-ik_\mu x^\mu}\,,$
$f(x) = \mathrm{Tr}\left(\hat{\mathcal{W}}[f]\hat{\Delta}(x)\right)\,,$	$\mathrm{Tr}\hat{\mathcal{W}}[f] = \int d^D x\, f(x)$

define derivation operator by $\quad [\hat{\partial}_\mu, \hat{x}^\nu] = \delta^\nu_\mu\,, \qquad [\hat{\partial}_\mu, \hat{\mathcal{W}}[f]] = \hat{\mathcal{W}}[\partial_\mu f]$

$$\hat{\mathcal{W}}[f]\hat{\mathcal{W}}[g] = \hat{\mathcal{W}}[f \star g]$$

Groenewold-Moyal space

definition of the Groenewold-Moyal *-product:

$$f(x) \star g(x) = \iint \frac{d^Dk \quad d^Dk'}{(2\pi)^D (2\pi)^D} \tilde{f}(k)\tilde{g}(k') e^{-\frac{i}{2}k_\mu \theta^{\mu\nu} k'_\nu} e^{-i(k_\mu + k'_\mu)x^\mu}$$

$$= e^{\frac{i}{2}\theta^{\mu\nu}\partial^x_\mu \partial^y_\nu} f(x) g(y)\Big|_{x=y} \neq g(x) \star f(x)$$

... or with more fields:

$$f_1(x) \star \cdots \star f_m(x) = \iiint \frac{d^Dk_1}{(2\pi)^D} \cdots \frac{d^Dk_m}{(2\pi)^D} e^{i\sum_{i=1}^m k_i x} \tilde{f}_1(k_1) \cdots \tilde{f}_m(k_m) e^{-\frac{i}{2}\sum_{i<j}^m k_i \theta k_j}$$

invariance under cyclic permutations of the integral

$$\int d^Dx\, f(x) \star g(x) \star h(x) = \int d^Dx h(x) \star f(x) \star g(x)$$

and

$$\frac{\delta}{\delta f_1(y)} \int d^Dx \left(f_1 \star f_2 \star \cdots \star f_m\right)(x) = \left(f_2 \star \cdots \star f_m\right)(y)$$

QFT ON DEFORMED SPACE-TIME

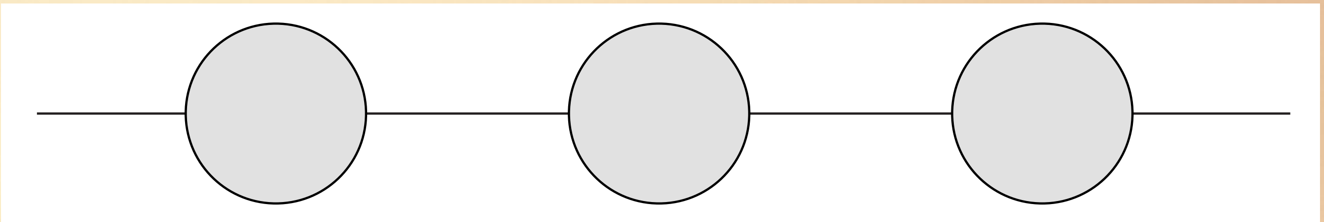
For a field theory in Euclidean space this means:
interaction vertices gain phases, whereas propagators remain unchanged, e.g.:

$$S = \mathrm{Tr}\left(\frac{1}{2}[\hat{\partial}_\mu, \hat{\mathcal{W}}[\phi]]^2 + \frac{m^2}{2}\hat{\mathcal{W}}[\phi]^2 + \frac{\lambda^2}{4!}\hat{\mathcal{W}}[\phi]^4\right)$$
$$= \int d^4x \left(\frac{1}{2}\partial_\mu\phi \star \partial^\mu\phi + \frac{m^2}{2}\phi\star\phi + \frac{\lambda}{4!}\phi\star\phi\star\phi\star\phi\right)$$

and some Feynman integrals ("non-planar diagrams") have phases which act as UV-regulators

$$\frac{1}{4}\int d^4k \frac{e^{ik\theta p}}{k^2+m^2} = \sqrt{\frac{m^2}{(\theta p)^2}} K_1\left(\sqrt{m^2(\theta p)^2}\right) \approx \frac{1}{(\theta p)^2} + c.m^2 \ln(\theta p)^2$$

⇨ ***origin of the UV/IR mixing problem***



Gauge fields on theta-deformed spaces

Star commutator of two Lie algebra valued functions:

$$[\alpha \stackrel{\star}{,} \beta] = \frac{1}{2}\{\alpha^a \stackrel{\star}{,} \beta^b\}[T^a, T^b] + \frac{1}{2}[\alpha^a \stackrel{\star}{,} \beta^b]\{T^a, T^b\}$$

→ *must always consider enveloping algebras, such as U(N), O(N) or USp(2N)*

Non-commutative Yang-Mills action:

$$\begin{aligned} S &= \frac{1}{4}\mathrm{Tr}\big([\hat{\partial}_\mu, \hat{\mathcal{W}}[A]_\nu] - [\hat{\partial}_\nu, \hat{\mathcal{W}}[A]_\mu] - ig[\hat{\mathcal{W}}[A]_\mu, \hat{\mathcal{W}}[A]_\nu]\big)^2 \\ &= \frac{1}{4}\int d^D x \, \mathrm{tr}_N \left(F_{\mu\nu}(x) \star F^{\mu\nu}(x)\right) \,, \end{aligned}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - \underline{ig[A_\mu \stackrel{\star}{,} A_\nu]}$$

It is invariant under the infinitesimal gauge transformations

$$\begin{aligned} \delta_\alpha A_\mu(x) &= D_\mu \alpha(x) = \partial_\mu A(x) - ig[A_\mu(x) \stackrel{\star}{,} \alpha(x)] \,, \\ \delta_\alpha F_{\mu\nu}(x) &= -ig[F_{\mu\nu}(x) \stackrel{\star}{,} \alpha(x)] \end{aligned}$$

Covariant coordinates & matrix models

star gauge transformation of a scalar field: $\phi(x) \to u(x) \star \phi(x) \star u(x)^\dagger$

In contrast to the commutative case, $x^\mu \star \phi(x)$ does not transform covariantly.

→ *define "covariant" coordinates:*

$$\tilde{X}_\mu := \tilde{x}_\mu + gA_\mu, \qquad \tilde{x}_\mu := \theta^{-1}_{\mu\nu} x^\nu$$

$$\tilde{X}_\mu \to u(x) \star \tilde{X}_\mu \star u(x)^\dagger\,,$$

$$\tilde{X}_\mu \phi(x) \to u(x) \star \tilde{X}_\mu \phi(x) \star u(x)^\dagger$$

$$[x^\mu \stackrel{\star}{,} f(x)] = i\theta^{\mu\nu}\partial_\nu f(x)\,,$$

$$i\,[\tilde{X}_\mu \stackrel{\star}{,} \tilde{X}_\nu] = \theta^{-1}_{\mu\nu} - gF_{\mu\nu}$$

NC Yang-Mills action:

$$\int d^4x F_{\mu\nu} \star F^{\mu\nu} \quad \to \quad -\frac{1}{g^2}\int d^4x\, [\tilde{X}_\mu \stackrel{\star}{,} \tilde{X}_\nu] \star [\tilde{X}^\mu \stackrel{\star}{,} \tilde{X}^\nu]$$

Yang-Mills matrix model:

$$S_{YM} = -\mathrm{Tr}[X^a, X^b][X^c, X^d]\eta_{ac}\eta_{bd}$$

Emergent gravity from matrix models

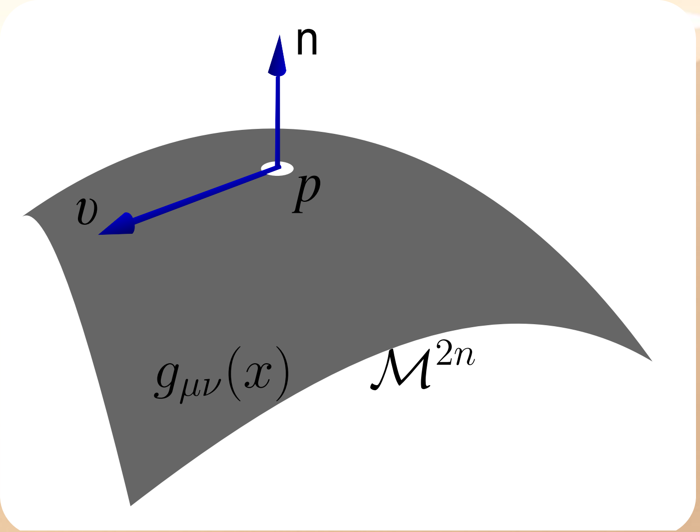
$$S_{YM} = -\mathrm{Tr}[X^a, X^b][X^c, X^d]\eta_{ac}\eta_{bd}$$

$$X^a = \left(X^\mu, \Phi^i\right), \ \ \mu = 1, \ldots, 2n, \ \ i = 1, \ldots, D-2n,$$

so that $\Phi^i(X) \sim \phi^i(x)$ define embedding $\mathcal{M}^{2n} \hookrightarrow \mathbf{R}^D$

$g_{\mu\nu}(x) = \partial_\mu x^a \partial_\nu x^b \eta_{ab}$ (in semi-classical limit)

$$G^{\mu\nu} = e^{-\sigma}\theta^{\mu\rho}\theta^{\nu\sigma}g_{\rho\sigma}, \qquad e^{-\sigma} \equiv \frac{\sqrt{\det\theta^{-1}_{\mu\nu}}}{\sqrt{\det G_{\rho\sigma}}}$$



$$\begin{aligned} S[\phi] &= -\mathrm{Tr}\left[X^a, \phi\right]\left[X^c, \phi\right]\eta_{ac} \\ &\sim \int d^4x \sqrt{\det\theta^{-1}_{\mu\nu}}\, \{x^a, \phi\}_{\mathrm{PB}}\{x^c, \phi\}_{\mathrm{PB}}\eta_{ac} \\ &= \int d^4x \sqrt{\det\theta^{-1}_{\mu\nu}}\, \theta^{\mu\nu}\partial_\mu x^a \partial_\nu\phi\, \theta^{\rho\sigma}\partial_\rho x^c \partial_\sigma\phi\, \eta_{ac} \\ &= \int d^4x \sqrt{\det G_{\mu\nu}}\, G^{\nu\sigma}\partial_\nu\phi\partial_\sigma\phi \end{aligned}$$

(cf. *Class.Quant.Grav.* **27** (2010) 133001)

Introducing the IKKT model

$$S_{\mathrm{IKKT}} = \mathrm{Tr}\left(\left[X^a, X^b\right]\left[X_a, X_b\right] + \bar{\Psi}\not{D}\Psi\right) ,$$
$$\not{D}\Psi := \gamma_a\left[X^a, \Psi\right] , \qquad \{\gamma_a, \gamma_b\} = 2\eta_{ab}$$

IKKT matrix model is supersymmetric and expected to be renormalizable - cf. *Nucl.Phys.* **B498** (1997) 467.

Majorana-Weyl spinor $\Psi = \mathcal{C}\bar{\Psi}^T$, is invariant under SUSY:

$$\delta^1\Psi = \frac{i}{4}[X^a, X^b][\gamma_a, \gamma_b]\epsilon , \qquad \delta^1 X^a = i\bar{\epsilon}\gamma^a\Psi$$
$$\delta^2\Psi = \xi , \qquad \delta^2 X^a = 0$$

Further symmetries:

$$X^a \to U^{-1}X^aU , \quad \Psi \to U^{-1}\Psi U , \quad U \in U(\mathcal{H}) , \quad \text{gauge inv.}$$
$$X^a \to \Lambda(g)^a_b X^b , \quad \Psi_\alpha \to \tilde{\pi}(g)^\beta_\alpha \Psi_\beta , \quad g \in \widetilde{SO}(D) , \quad \text{rotations,}$$
$$X^a \to X^a + c^a\mathbb{1} , \quad c^a \in \mathbb{R} , \quad \text{translations}$$

The IKKT model

$$S_{\mathrm{IKKT}} = \mathrm{Tr}\left(\left[X^a, X^b\right]\left[X_a, X_b\right] + \bar{\Psi}\gamma_a\left[X^a, \Psi\right]\right)$$

- Originally proposed as non-perturbative definition of type IIB string theory,
- Seems to provide a good candidate for quantum gravity and other fundamental interactions, i.e.

$$X^a = \begin{pmatrix} \bar{X}^\mu - \theta^{\mu\nu} A_\nu(\bar{X}^\mu) \\ \Phi^i(\bar{X}^\mu) \end{pmatrix}$$

- Here, we consider general NC brane configurations and their effective gravity in the matrix model,
- Assume soft breaking of SUSY below some scale Λ and compute the effective action using a Heatkernel expansion.

The fermionic action

$$S_\Psi = \mathrm{Tr}\Psi^\dagger \not{D}\Psi = \mathrm{Tr}\Psi^\dagger \gamma_a [X^a, \Psi]$$
$$e^{-\Gamma[X]} = \int d\Psi d\Psi^\dagger e^{-S_\Psi} = (\mathrm{const.}) \exp\left(\frac{1}{2}\mathrm{Tr}\log(\not{D}^2)\right)$$
$$\not{D}^2\Psi = \gamma_a\gamma_b[X^a,[X^b,\Psi]] = (\not{D}_0^2 + V)\Psi$$

- Consider fermions coupled to NC background
- Matrices X^a: perturbations around Moyal quantum plane
  ⇒ introduce NC scale $\Lambda_{\mathrm{NC}}^4 = e^{-\sigma}$

$$[\bar{X}^\mu, \bar{X}^\nu] = i\bar{\theta}^{\mu\nu} = i\Lambda_{\mathrm{NC}}^{-2}\begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix}$$
$$X^\mu = (\bar{X}^\mu + \mathcal{A}^\mu, \phi^i) = (\bar{X}^\mu - \bar{\theta}^{\mu\nu}A_\nu, \Lambda_{NC}^2\varphi^i)$$

HEATKERNEL EXPANSION

Consider a Duhamel expansion:

$$\frac{1}{2}\mathrm{Tr}\left(\log \not{D}^2 - \log \not{D}_0^2\right) \rightarrow -\frac{1}{2}\mathrm{Tr}\int_0^\infty \frac{d\alpha}{\alpha}\left(e^{-\alpha\not{D}^2} - e^{-\alpha\not{D}_0^2}\right) e^{-\frac{\Lambda_{\mathrm{NC}}^4}{\alpha\Lambda^2}}$$

$$= \Lambda^4 \sum_{n\geq 0}\int d^4x\, \mathcal{O}\left(\frac{(p, A, \varphi)^n}{(\Lambda, \Lambda_{\mathrm{NC}})^n}\right)$$

In contrast to previous work, we consider a „semi-classical“ low energy regime characterized by

$$\epsilon(p) := p^2\Lambda^2/\Lambda_{NC}^4 \ll 1$$

Can expand UV/IR mixing terms as

$$e^{-p^2\Lambda_{\mathrm{NC}}^4/\alpha} \approx \sum_{m\geq 0} a_m \epsilon(p)^m$$

Avoids pathological phenomena which would appear if ~~$\Lambda \to \infty$~~ and Λ_{NC} fixed

Expansion in 3 small parameters: $\Gamma \sim \Lambda^4 \sum_{n,l,k\geq 0}\int d^4x\, \mathcal{O}\left(\epsilon(p)^n (\frac{p^2}{\Lambda_{\mathrm{NC}}^2})^l (\frac{p^2}{\Lambda^2})^k\right)$

Gauge invariance of effective NCGFT

Adding up the first 3 order contributions leads to the following order Λ^4 terms:

$$\Gamma_{\Lambda^4}(A,\varphi,p) = \frac{\Lambda^4 \mathrm{Tr}\mathbb{1}}{16\Lambda_{\mathrm{NC}}^4}\int \frac{d^4x}{(2\pi)^2}\sqrt{g}\Big(g^{\alpha\beta}D_\alpha\varphi^i D_\beta\varphi_i - \frac{1}{2}\Lambda_{\mathrm{NC}}^4\big(\bar\theta^{\mu\nu}F_{\nu\mu}\bar\theta^{\rho\sigma}F_{\sigma\rho} + (\bar\theta^{\sigma\sigma'}F_{\sigma\sigma'})(F\bar\theta F\bar\theta)\big) - 2\bar\theta^{\nu\mu}F_{\mu\alpha}g^{\alpha\beta}\partial_\nu\varphi^i\partial_\beta\varphi_i + \frac{1}{2}(\bar\theta^{\mu\nu}F_{\mu\nu})g^{\alpha\beta}\partial_\beta\varphi^i\partial_\alpha\varphi_i + \mathrm{h.o.}\Big)$$

➡ These terms are manifestly gauge invariant

Free contribution:

$$\Gamma[\bar X] = -\frac{1}{2}\mathrm{Tr}\int_0^\infty \frac{d\alpha}{\alpha}e^{-\alpha \not{D}_0^2 - \frac{\Lambda_{\mathrm{NC}}^4}{\alpha\Lambda^2}} = -\frac{\Lambda^4\mathrm{Tr}\mathbb{1}}{8}\int\frac{d^4x}{(2\pi)^2}\sqrt{g}$$

➡ Along with general geometrical considerations, this suffices to predict some loop computations

SO(D) INVARIANCE OF GENERALIZED MM

Can reproduce gauge sector of induced result by a semi-classical analysis with vanishing embedding fields:

$$\frac{1}{\sqrt{\frac{1}{2}(\mathrm{Tr}J^2)^2 - \mathrm{Tr}J^4}}\Bigg|_{\partial\phi^i=0} \sim \frac{\Lambda_{\mathrm{NC}}^4}{2}\Bigg(1 + \frac{1}{2}\bar{\theta}^{\mu\nu}F_{\mu\nu} + \frac{1}{4}(\bar{\theta}F)^2 + \frac{1}{4}(\bar{\theta}F)(F\bar{\theta}F\bar{\theta}) + \mathcal{O}(F^4)\Bigg)$$

Effective action can be written as a generalized matrix model with manifest SO(D) symmetry.

Can be further generalized to include curvature terms, e.g.:

$$\int \frac{d^4x}{(2\pi)^2}\sqrt{g}\,\Lambda(x)^2\Big(R + (\bar{\Lambda}_{\mathrm{NC}}^4 e^{-\sigma}\theta^{\mu\rho}\theta^{\eta\alpha}R_{\mu\rho\eta\alpha} - 4R) + c'\partial^\mu\sigma\partial_\mu\sigma\Big)$$

Such terms appear in the semiclassical limit of higher order matrix terms.

Conclusion

- Have discussed properties and problems (such as UV/IR mixing) of non-commutative quantum field theories.
- Introduced matrix models, especially the IKKT model and its properties, such as emergent gravity.
- Computed the effective fermion action, first from NC field theory, then from the matrix model point of view.
- Many interesting open questions.

References

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Thank you for your attention!