Geometry as a Semiclassical Effect in a Quantum World - Emergent gravity from matrix models

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$[x^\mu, x^\nu] = i\Theta^{\mu\nu}$

Background:

Motivation

- incompatibility between GR and QFT:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \langle T_{\mu\nu} \rangle \ ,$$

l.h.s is the classical Einstein tensor, whereas r.h.s. is the expectation value of a quantum mechanical operator: the energy-momentum tensor $T_{\mu\nu}$

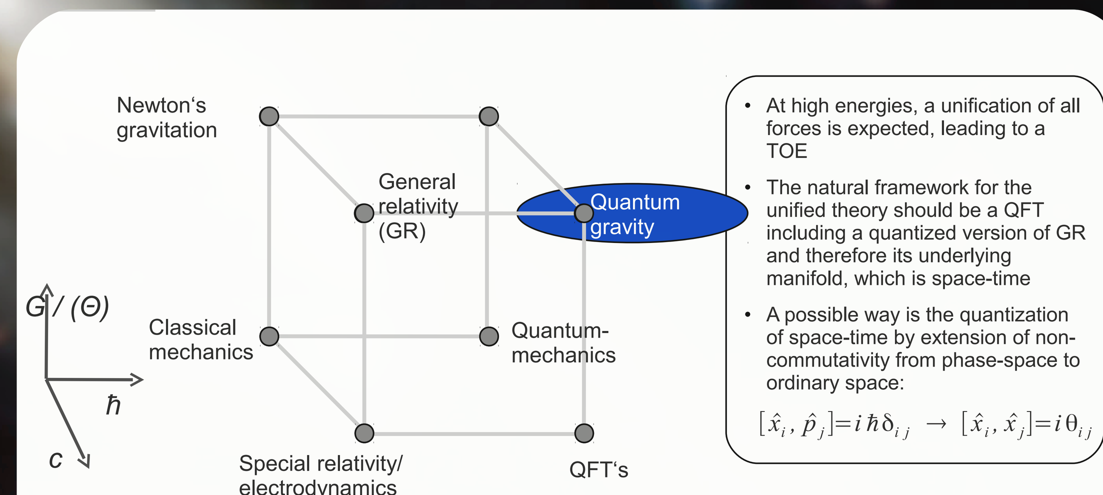
- natural limit in experimental length resolution: better length resolution requires higher energy, energy required for resolution of the Planck length has a Schwarzschild radius of the Planck length

$$\Delta x_\mu \simeq \lambda_p = \sqrt{\frac{G\hbar}{c^3}} \simeq 10^{-33} cm$$

$\rightarrow$ hint towards quantized space-time

History

- H. Snyder (1946/47): "minimal length" to smear out point-like interactions as UV regularization in QFT
- early 1990s: J. Madore, A. Connes, T. Filk and others (fuzzy sphere, extensive study of non-commutative geometry, new IR divergences $\rightarrow$ UV/IR mixing)
- N. Seiberg & E. Witten (1999): connection to string theory (effective low-energy action on D-branes with strong B-field background, non-commutativity parameter related to B-field as $\theta \sim B^{-1}$)



A possible way of quantizing space-time is the introduction of non-commutative space-time coordinates

QFT on theta-deformed space-time

- consider non-commuting coordinates

$$[\hat{x}^\mu, \hat{x}^\nu] = i\theta^{\mu\nu} \qquad \Rightarrow \qquad \text{implies uncertainty relation:}$$
$$\Delta x^\mu \Delta x^\nu \geq \tfrac{1}{2}|\theta^{\mu\nu}| \sim (\lambda_p)^2 \,,$$

where $\lambda_p \approx 10^{-33}$cm is the Planck length

- cp. Heisenberg uncertainty:

$$[\hat{x}, \hat{p}] = i\hbar \qquad \leftrightarrow \qquad \Delta x \Delta p \geq \tfrac{1}{2}\hbar$$

- definition of the Groenewold-Moyal $\star$-product:

$$f(x) \star g(x) = e^{\frac{i}{2}\theta^{\mu\nu}\partial^x_\mu \partial^y_\nu} f(x) g(y)\Big|_{x=y} \neq g(x) \star f(x)$$

$\Rightarrow$ can use regular coordinates x instead of operators $\hat{x}$, since:

$$[x^\mu \stackrel{\star}{,} x^\nu] = i\theta^{\mu\nu}$$

- QFT: interaction vertices gain phases, whereas propagators remain unchanged
- some Feynman integrals ("non-planar diagrams") have phases, e.g.

$$\int d^4k \frac{e^{ik\tilde{p}}}{k^2 + i\epsilon} \propto \frac{1}{\tilde{p}^2} \quad \text{with} \quad \tilde{p}^\mu = \theta^{\mu\nu} p_\nu$$

- phases act as UV-regulators,

$\Rightarrow$ origin of the *UV/IR mixing* problem

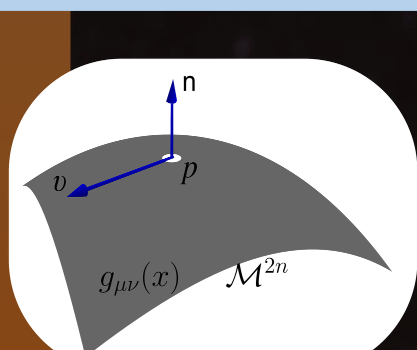
Emergent gravity

$$S_{YM} = -\mathrm{Tr}[X^a, X^b][X^c, X^d]\eta_{ac}\eta_{bd}$$

$X^a = \left(X^\mu, \Phi^i\right)$, $\mu = 1, \ldots, 2n$, $i = 1, \ldots, D - 2n$,
so that $\Phi^i(X) \sim \phi^i(x)$ define embedding $\mathcal{M}^{2n} \hookrightarrow \mathbf{R}^D$
$g_{\mu\nu}(x) = \partial_\mu x^a \partial_\nu x^b \eta_{ab}$ (in semi-classical limit)

$$G^{\mu\nu} = e^{-\sigma}\theta^{\mu\rho}\theta^{\nu\sigma} g_{\rho\sigma}, \qquad e^{-\sigma} \equiv \frac{\sqrt{\det \theta^{-1}_{\mu\nu}}}{\sqrt{\det G_{\rho\sigma}}}$$

$$\begin{aligned} S[\phi] &= -\mathrm{Tr}\,[X^a, \phi]\,[X^c, \phi]\,\eta_{ac} \\ &\sim \int d^4x \sqrt{\det \theta^{-1}_{\mu\nu}}\, \{x^a, \phi\}_{\rm PB} \{x^c, \phi\}_{\rm PB} \eta_{ac} \\ &= \int d^4x \sqrt{\det \theta^{-1}_{\mu\nu}}\, \theta^{\mu\nu}\partial_\mu x^a \partial_\nu \phi\, \theta^{\rho\sigma}\partial_\rho x^c \partial_\sigma \phi\, \eta_{ac} \\ &= \int d^4x \sqrt{\det G_{\mu\nu}}\, G^{\nu\sigma}\partial_\nu\phi\partial_\sigma\phi \end{aligned}$$



Covariant coordinates & matrix models

star gauge transformation of a scalar field: $\phi(x) \rightarrow u(x) \star \phi(x) \star u(x)^\dagger$
In contrast to the commutative case, $x^\mu \star \phi(x)$ does not transform covariantly.

define "covariant" coordinates:

$$\tilde{X}_\mu := \tilde{x}_\mu + gA_\mu, \qquad \tilde{x}_\mu := \theta^{-1}_{\mu\nu} x^\nu$$
$$\tilde{X}_\mu \rightarrow u(x) \star \tilde{X}_\mu \star u(x)^\dagger \,,$$
$$\tilde{X}_\mu \phi(x) \rightarrow u(x) \star \tilde{X}_\mu \phi(x) \star u(x)^\dagger$$

$$i\left[\tilde{X}_\mu \stackrel{\star}{,} \tilde{X}_\nu\right] = \theta^{-1}_{\mu\nu} - gF_{\mu\nu}$$

NC Yang-Mills action: $\int d^4x F_{\mu\nu} \star F^{\mu\nu} \rightarrow -\frac{1}{g^2}\int d^4x \left[\tilde{X}_\mu \stackrel{\star}{,} \tilde{X}_\nu\right] \star \left[\tilde{X}^\mu \stackrel{\star}{,} \tilde{X}^\nu\right]$

Yang-Mills matrix model: $S_{YM} = -\mathrm{Tr}[X^a, X^b][X^c, X^d]\eta_{ac}\eta_{bd}$

UV/IR mixing

"naïve" models such as

$$S_{\phi 4} = \int d^4x \left[\frac{1}{2}\left(\partial^\mu\phi \star \partial_\mu\phi + m^2 \phi \star \phi\right) + \frac{\lambda}{4!}\phi^{\star 4}\right],$$
$$S_{\rm YM} = -\frac{1}{4}\int d^4x F_{\mu\nu} \star F^{\mu\nu} + \ldots$$

with $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig\,[A_\mu \stackrel{\star}{,} A_\nu]$
lead to (gauge independent) IR singular self-energy graphs

$$\Pi^{\mu\nu}_{\phi 4,\rm IR}(p) \propto \frac{1}{\tilde{p}^2}, \qquad \Pi^{\mu\nu}_{\rm YM,IR}(p) \propto \frac{\tilde{p}^\mu \tilde{p}^\nu}{(\tilde{p}^2)^2} \quad \text{with} \quad \tilde{p}^\mu = \theta^{\mu\nu}p_\nu$$

$\Rightarrow$ Graphs with these insertions are IR divergent!

Renormalizable models

- the Grosse-Wulkenhaar model (2003), where the ϕ^4 theory was supplemented by a (translation-invariance breaking) oscillator term ($\approx \tilde{x}^2\phi^2$),
- and a translation-invariant model by Gurau, Magnen, Rivasseau and Tanasa (2008), where a $\phi(-p)\frac{a^2}{\tilde{p}^2}\phi(p)$ term was added.

Dimensions of the Universe:

Planck length	$l_p \sim 10^{-35}$m
accessible by LHC	$\sim 10^{-20}$m (or 10TeV)
radius of an atom	$\sim 10^{-10}$m
earth's diameter	$\sim 10^{7}$m
radius of the visible universe	$\sim 10^{10}$LY (or $10^{61} l_p$)

"Terra incognita" | Physically accessible

The IKKT model:

Introducing the IKKT model

$$S_{\rm IKKT} = \mathrm{Tr}\left(\left[X^a, X^b\right][X_a, X_b] + \bar{\Psi}\not{D}\Psi\right),$$
$$\not{D}\Psi := \gamma_a\,[X^a, \Psi]\,, \qquad \{\gamma_a, \gamma_b\} = 2\eta_{ab}$$

IKKT matrix model is supersymmetric and expected to be renormalizable — cf. *Nucl.Phys.* **B498** (1997) 467.

Majorana-Weyl spinor $\Psi = \mathcal{C}\bar{\Psi}^T$, is invariant under SUSY:

$$\delta^1\Psi = \frac{i}{4}[X^a, X^b][\gamma_a, \gamma_b]\epsilon\,, \qquad \delta^1 X^a = i\bar{\epsilon}\gamma^a\Psi$$
$$\delta^2\Psi = \xi\,, \qquad \delta^2 X^a = 0$$

Further symmetries:

$$\begin{aligned} &X^a \rightarrow U^{-1}X^aU\,, && \Psi \rightarrow U^{-1}\Psi U\,, && U \in U(\mathcal{H})\,, && \text{gauge inv.} \\ &X^a \rightarrow \Lambda(g)^a_b X^b\,, && \Psi_\alpha \rightarrow \tilde{\pi}(g)^\beta_\alpha \Psi_\beta\,, && g \in \widetilde{SO}(D)\,, && \text{rotations,} \\ &X^a \rightarrow X^a + c^a\mathbb{1}\,, && && c^a \in \mathbb{R}\,, && \text{translations} \end{aligned}$$

- Originally proposed as non-perturbative definition of type IIB string theory,
- Seems to provide a good candidate for quantum gravity and other fundamental interactions, i.e.

$$X^a = \begin{pmatrix} \bar{X}^\mu - \theta^{\mu\nu}A_\nu(\bar{X}^\mu) \\ \Phi^i(\bar{X}^\mu) \end{pmatrix}$$

- Here, we consider general NC brane configurations and their effective gravity in the matrix model,
- Assume soft breaking of SUSY below some scale Λ and compute the effective action using a Heatkernel expansion.

Example of curvature terms appearing semiclassically in the model:

$$\int \frac{d^4x}{(2\pi)^2}\sqrt{g}\,\Lambda(x)^2\Big(R + (\bar{\Lambda}^4_{\rm NC}e^{-\sigma}\theta^{\mu\rho}\theta^{\eta\alpha}R_{\mu\rho\eta\alpha} - 4R) + c'\partial^\mu\sigma\partial_\mu\sigma\Big)$$

The fermionic action

$$S_\Psi = \mathrm{Tr}\Psi^\dagger\not{D}\Psi = \mathrm{Tr}\Psi^\dagger\gamma_a[X^a, \Psi]$$
$$e^{-\Gamma[X]} = \int d\Psi d\Psi^\dagger e^{-S_\Psi} = (\text{const.})\exp\left(\frac{1}{2}\mathrm{Tr}\log(\not{D}^2)\right)$$
$$\not{D}^2\Psi = \gamma_a\gamma_b[X^a, [X^b, \Psi]] = (\not{D}_0^2 + V)\Psi$$

- Consider fermions coupled to NC background
- Matrices X^a: perturbations around Moyal quantum plane
  introduce NC scale $\Lambda^4_{\rm NC} = e^{-\sigma}$

$$[\bar{X}^\mu, \bar{X}^\nu] = i\bar{\theta}^{\mu\nu} = i\Lambda^{-2}_{\rm NC}\begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix}$$
$$X^\mu = (\bar{X}^\mu + \mathcal{A}^\mu, \phi^i) = (\bar{X}^\mu - \bar{\theta}^{\mu\nu}A_\nu, \Lambda^2_{NC}\varphi^i)$$

Small parameters of expansion

In contrast to previous work, we consider a "semi-classical" low energy regime characterized by

$$\epsilon(p) := p^2\Lambda^2/\Lambda^4_{NC} \ll 1$$

Can expand UV/IR mixing terms as

$$e^{-p^2\Lambda^4_{\rm NC}/\alpha} \approx \sum_{m\geq 0} a_m \epsilon(p)^m$$

Avoids pathological phenomena which would appear if
~~$\Lambda \rightarrow \infty$~~ and $\Lambda_{\rm NC}$ fixed

Expansion in 3 small parameters:

$$\Gamma \sim \Lambda^4 \sum_{n,l,k\geq 0}\int d^4x\, \mathcal{O}\left(\epsilon(p)^n\left(\frac{p^2}{\Lambda^2_{\rm NC}}\right)^l\left(\frac{p^2}{\Lambda^2}\right)^k\right)$$

Selection of references

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