Renormalization of Quantum Field Theories on Non-Commutative Spaces

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Motivation

- incompatibility between GR and QFT:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \langle T_{\mu\nu} \rangle$$

lhs: classical Einstein tensor, rhs: ev of an operator

→ ***hint towards quantized space-time***

- natural limit in experimental length resolution: better length resolution requires higher energy, energy required for resolution of the Planck length has a Schwarzschild radius of the Planck length

$$\Delta x_\mu \simeq \lambda_p = \sqrt{\frac{G\hbar}{c^3}} \simeq 10^{-33}\mathrm{cm}$$

Groenewold-Moyal space

- assume non-commuting space-time coordinates:
$[\hat{x}^\mu, \hat{x}^\nu] = i\theta^{\mu\nu}$, $\Rightarrow$ leads to uncertainty relation
$$\Delta x^\mu \Delta x^\nu \geq \frac{1}{2}|\theta^{\mu\nu}| \sim (\lambda_p)^2$$
- exists isomorphism mapping between NC algebra and commutative one, e.g. Weyl map
$$W : \mathcal{A} \to \widehat{\mathcal{A}}, \qquad x^i \mapsto \hat{x}^i$$
- definition of the Groenewold-Moyal $*$-product:
$$f(x) \star g(x) = e^{\frac{i}{2}\theta^{\mu\nu}\partial^x_\mu \partial^y_\nu} f(x)g(y)\Big|_{x=y} \neq g(x) \star f(x)$$
- invariance under cyclic permutations of the integral
$$\int d^4x f(x) \star g(x) \star h(x) = \int d^4x h(x) \star f(x) \star g(x)$$

QFT on deformed space-time

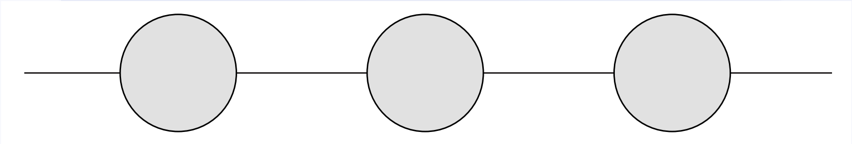
For a field theory this means:

- interaction vertices gain phases, whereas propagators remain unchanged,

- and some Feynman integrals (”non-planar diagrams”) have phases which act as UV-regulators

$$\frac{1}{4}\int d^4k \frac{e^{ik\theta p}}{k^2+m^2} = \sqrt{\frac{m^2}{(\theta p)^2}} K_1\left(\sqrt{m^2(\theta p)^2}\right) \approx \frac{1}{(\theta p)^2} + c.m^2 \ln(\theta p)^2$$

⇨ ***origin of the UV/IR mixing problem***



UV/IR mixing

"naive" models such as

$$S_{\mathrm{sc}} = \int d^4x \left[\frac{1}{2} \left(\partial^\mu \phi \star \partial_\mu \phi + m^2 \phi \star \phi \right) + \frac{\lambda}{4!} \phi^{\star 4} \right],$$

$$S_{\mathrm{gf}} = -\frac{1}{4} \int d^4x F_{\mu\nu} \star F^{\mu\nu} + \dots$$

$$\text{with} \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu \stackrel{\star}{,} A_\nu]$$

lead to IR singular self-energy graphs

$$\Pi^{\mu\nu}_{\mathrm{sc,IR}}(p) \propto \frac{1}{(\theta p)^2}, \qquad \Pi^{\mu\nu}_{\mathrm{gf,IR}}(p) \propto \frac{(\theta p)^\mu (\theta p)^\nu}{((\theta p)^2)^2}$$

Graphs with these insertions are IR divergent!

Previous successes

So far, there are two models for scalar field theories where the UV/IR mixing problem could be solved by adding additional terms in the action:

- the Grosse-Wulkenhaar model (2003), where the ϕ^4 theory was supplemented by a (translation-invariance breaking) oscillator term ($\approx \{x \stackrel{\star}{,} \phi\}^{\star 2}$),
- and a translation-invariant model by Gurau, Magnen, Rivasseau and Tanasa (2008), where a $\phi(-p)\frac{1}{(\theta p)^2}\phi(p)$ term was added.

Both models could be proved to be renormalizable to all orders.

Scalar NCQFT

Action introduced by Gurau et. al. in
Commun.Math.Phys. **287** (2009) 275:

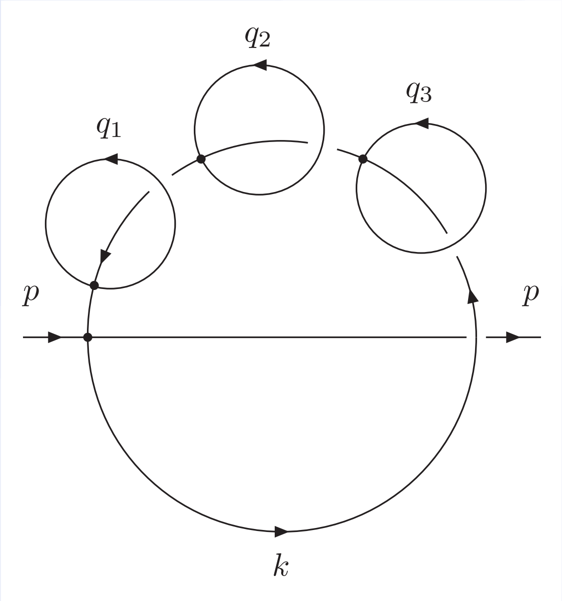
$$\mathcal{S} = \int d^4x \left[\frac{1}{2} \left(\partial^\mu \phi \star \partial_\mu \phi + m^2 \phi \star \phi - \phi \star \frac{a^2}{\widetilde{\square}} \phi \right) + \frac{\lambda}{4!} \phi^{\star 4} \right]$$

Propagator: $G(k) = \dfrac{1}{k^2 + m^2 + \frac{a^2}{k^2}}$

$\Rightarrow$ new "damping" behaviour $\lim_{k \to 0} G(k) = 0$

Damping in higher loops

$$\Pi^{\rm npl}_{\rm IR}(k) \propto \frac{1}{(\theta k)^2}$$



$$\Pi^{n\text{npl-ins.}}(p) \equiv \lambda^2 \int d^4k \, \frac{e^{ik\theta p}}{((\theta k)^2)^n \left[k^2 + m^2 + \frac{a^2}{(\theta k)^2}\right]^{n+1}}$$

- $a = 0$: IR divergence for $n \geq 2$, i.e. integrand $\sim (k^2)^{-n}$
- $a \neq 0$: finite, i.e. integrand $\sim \dfrac{1}{((\theta k)^2)^n \left[\frac{a^2}{(\theta k)^2}\right]^{n+1}} = \dfrac{(\theta k)^2}{(a^2)^{n+1}}$

Construct a gauge field model

$$U_\star(1): \ \delta_\varepsilon A_\mu = \partial_\mu \varepsilon + ig[A_\mu \stackrel{\star}{,} \varepsilon]\,,$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu \stackrel{\star}{,} A_\nu]$$

$$D_\mu \bullet \equiv \partial_\mu \bullet - ig[A_\mu \stackrel{\star}{,} \bullet] \qquad \widetilde{D}_\mu = \theta_{\mu\nu} D^\nu$$

$$\int d^4x\, \phi(x) \frac{a^2}{\theta^2 \Box} \phi(x) \quad \Rightarrow \quad \boxed{\int d^4x\, \frac{1}{4} F^{\mu\nu} \star \frac{a^2}{D^2 \widetilde{D}^2} \star F_{\mu\nu}}$$

$$\delta_\varepsilon \left(\frac{1}{D^2} F \right) = ig[\varepsilon \stackrel{\star}{,} \frac{1}{D^2} F]$$

Drawback: infinite number of vertices ...

IR modified gauge field model

Model requires modification in the IR

→ Proposition:
Use techniques known from the Gribov-Zwanziger action in QCD

$$\int d^4x \, \frac{1}{4} F^{\mu\nu} \star \frac{a^2}{D^2 \widetilde{D}^2} \star F_{\mu\nu}$$

$$\gamma^4 g^2 \int d^4x \, f^{abc} A^b_\mu (\mathcal{M}^{-1})^{ad} f^{dec} A^e_\mu$$

New gauge field action

$$\mathcal{S}_{\text{loc}} = \int d^4x \Bigg[\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + s\left((\bar{Q}_{\mu\nu\alpha\beta} B_{\mu\nu} + h.c.) \frac{1}{\widetilde{\square}} \left(f_{\alpha\beta} + \frac{\sigma\theta_{\alpha\beta}}{2} \tilde{f} \right) \right)$$

$$+ s(\bar{\psi}^{\mu\nu} B_{\mu\nu}) + s\left(Q' \{ A_\mu \stackrel{\star}{,} A_\nu \} \frac{\tilde{\partial}_\mu \tilde{\partial}_\nu \tilde{\partial}_\rho}{\widetilde{\square}^2} A_\rho \right) + s(\bar{c}\partial_\mu A^\mu) \Bigg]$$

$sA_\mu = D_\mu c\,,$	$sc = igc \star c\,,$	$s\bar{c} = b\,,$	$sb = 0\,,$
$s\,\bar{\psi}_{\mu\nu} = \bar{B}_{\mu\nu}\,,$	$s\,\bar{B}_{\mu\nu} = 0\,,$	$s\,B_{\mu\nu} = \psi_{\mu\nu}\,,$	$s\psi_{\mu\nu} = 0\,,$
$s\,\bar{Q}_{\mu\nu\alpha\beta} = \bar{J}_{\mu\nu\alpha\beta}\,,$	$s\,\bar{J}_{\mu\nu\alpha\beta} = 0\,,$		
$s\,Q_{\mu\nu\alpha\beta} = J_{\mu\nu\alpha\beta}\,,$	$s\,J_{\mu\nu\alpha\beta} = 0\,,$	$s\,Q' = J'\,,$	$s\,J' = 0\,.$

Soft breaking

$$\bar{Q}_{\mu\nu\alpha\beta}\big|_{\rm phys} = Q_{\mu\nu\alpha\beta}\big|_{\rm phys} = Q'\big|_{\rm phys} = 0\,, \qquad J'\big|_{\rm phys} = ig\gamma'^2\,,$$

$$\bar{J}_{\mu\nu\alpha\beta}\big|_{\rm phys} = J_{\mu\nu\alpha\beta}\big|_{\rm phys} = \frac{\gamma^2}{4}\left(\delta_{\mu\alpha}\delta_{\nu\beta} - \delta_{\mu\beta}\delta_{\nu\alpha}\right)$$

⇓ Propagator with IR damping

$$G^{AA}_{\mu\nu}(k) = \boxed{\frac{1}{k^2\left(1+\frac{\gamma^4}{((\theta k)^2)^2}\right)}\left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right.} - f(\gamma,\sigma,k^2)\frac{(\theta k)_\mu(\theta k)_\nu}{(\theta k)^2}\Bigg)$$

? *Can we derive a “horizon condition” for the Gribov-like parameters?*

Conclusion

- Have constructed a promising candidate for a renormalizable NC gauge field model, but need to prove renormalizability to all orders.
- Method used to prove renormalizability of scalar models would break gauge invariance in our case.
- Is there a relation between UV/IR mixing and the Gribov problem in NC gauge field theories?

References

1. D. N. Blaschke, E. Kronberger, R. I. P. Sedmik and M. Wohlgenannt, Gauge Theories on Deformed Spaces, *SIGMA* **6** (2010) 062, [arXiv:1004.2127].

2. D. N. Blaschke, A. Rofner, R. I. P. Sedmik and M. Wohlgenannt, On Non-Commutative U*(1) Gauge Models and Renormalizability, *J. Phys.* **A43** (2010) 425401, [arXiv:0912.2634].

3. D. N. Blaschke, A New Approach to Non-Commutative U(N) Gauge Fields, *EPL* **91** (2010) 11001, [arXiv:1005.1578].

Thank you for your attention!

Backup slides

NCQFT & matrix models

Action of renormalizable scalar Grosse-Wulkenhaar model:

$$\mathcal{S}_{\mathrm{GW}} = \int d^4x \left(-\frac{1}{2}[\tilde{x}_\mu \stackrel{\star}{,} \phi]^{\star 2} - \frac{\Omega^2}{2}\{\tilde{x}_\mu \stackrel{\star}{,} \phi\}^{\star 2} - \frac{m^2}{2}\phi^{\star 2} + \frac{\lambda}{4!}\phi^{\star 4} \right)$$

where $[\tilde{x}_\mu \stackrel{\star}{,} \phi] = [(\theta^{-1}x)_\mu \stackrel{\star}{,} \phi] = i\partial_\mu \phi$

couple to gauge fields

$$[X_\mu \stackrel{\star}{,} \phi] \qquad \text{with } X_\mu = \tilde{x}_\mu + gA_\mu \,, \quad i[X_\mu \stackrel{\star}{,} X_\nu] = \theta^{-1}_{\mu\nu} - gF_{\mu\nu}$$

$$\mathcal{S}_{\mathrm{IKKT}} = \mathrm{Tr}\left([X^a, X^b][X_a, X_b] + \bar{\Psi}\gamma_a [X^a, \Psi] \right)$$

Supersymmetric IKKT matrix model is expected to be renormalizable - cf. *Nucl.Phys.* **B498** (1997) 467.

Emergent gravity

X^a Herm. matrices on $\mathcal{H}$, and η_{ab} is D-dim. flat metric

$$X^a = \left(X^\mu, \Phi^i\right), \ \mu = 1, \ldots, 2n, \ i = 1, \ldots, D-2n,$$

so that $\Phi^i(X) \sim \phi^i(x)$ define embedding $\mathcal{M}^{2n} \hookrightarrow \mathbf{R}^D$

$g_{\mu\nu}(x) = \partial_\mu x^a \partial_\nu x^b \eta_{ab}$ (in semi-classical limit)

$\mathcal{M}^{2n}$ endowed with a Poisson structure

$-i[X^\mu, X^\nu] \sim \{x^\mu, x^\nu\}_{pb} = \theta^{\mu\nu}(x) \Rightarrow$ "effective" metric

$$G^{\mu\nu} = e^{-\sigma}\theta^{\mu\rho}\theta^{\nu\sigma} g_{\rho\sigma}, \qquad e^{-\sigma} \equiv \frac{\sqrt{\det \theta^{-1}_{\mu\nu}}}{\sqrt{\det G_{\rho\sigma}}}$$

$$-\mathrm{Tr}[X^a, \Phi][X_a, \Phi] \sim \int d^4x \sqrt{\det \theta^{-1}}\, \theta^{\mu\nu}\theta^{\rho\sigma}\partial_\mu x^a \partial_\rho x_a \partial_\nu \phi \partial_\sigma \phi$$

(cf. Class.Quant.Grav. **27** (2010) 133001 [arXiv:1003.4134])

UV/IR mixing due to gravity

- add U(N) valued gauge fields: $X^\mu = \bar{X}^\mu + \mathcal{A}^\mu$
- Effective matrix model action then describes gauge fields in a gravitational background
- However, the U(1) and SU(N) subsectors play very different roles: U(1) purely gravitational

⇒ non-commutative U(N) gauge field theory describes SU(N) fields coupled to gravity

⇒ alternative interpretation of UV/IR mixing

(*cf. Class.Quant.Grav.* **27** (2010) 133001 [arXiv:1003.4134])