Heat Kernel Expansion and Induced Action for Matrix Models

Talk presented by Daniel N. Blaschke

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Matrix models of Yang-Mills type

$$S_{YM} = -\mathrm{Tr}[X^a, X^b][X^c, X^d]\eta_{ac}\eta_{bd}$$

X^a Herm. matrices on $\mathcal{H}$, and η_{ab} is D-dim. flat metric

$$X^a = \left(X^\mu, \Phi^i\right), \;\; \mu = 1, \ldots, 2n, \;\; i = 1, \ldots, D - 2n,$$

so that $\Phi^i(X) \sim \phi^i(x)$ define embedding $\mathcal{M}^{2n} \hookrightarrow \mathbf{R}^D$
$g_{\mu\nu}(x) = \partial_\mu x^a \partial_\nu x^b \eta_{ab}$ (in semi-classical limit)

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$\mathcal{M}^{2n}$ endowed with a Poisson structure

$-i[X^\mu, X^\nu] \sim \{x^\mu, x^\nu\}_{pb} = \theta^{\mu\nu}(x) \Rightarrow$ "effective" metric

$$G^{\mu\nu} = e^{-\sigma}\theta^{\mu\rho}\theta^{\nu\sigma}g_{\rho\sigma} = -(\mathcal{J}^2)^\mu_\rho g^{\rho\nu}, \qquad e^{-\sigma} \equiv \frac{\sqrt{\det \theta^{-1}_{\mu\nu}}}{\sqrt{\det G_{\rho\sigma}}}$$

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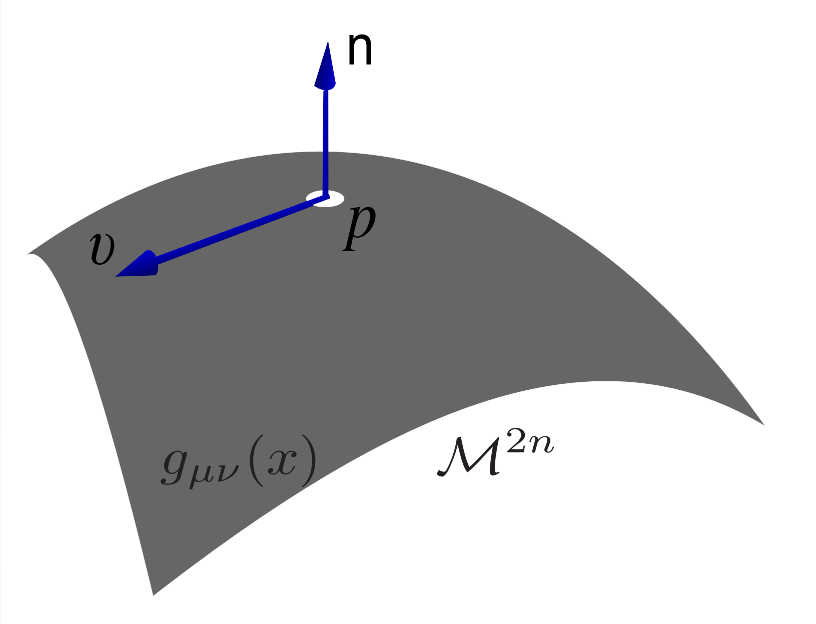
$$-\mathrm{Tr}[X^a, \Phi][X_a, \Phi] \sim \int d^4x \sqrt{\det \theta^{-1}}\, \theta^{\mu\nu}\theta^{\rho\sigma}\partial_\mu x^a \partial_\rho x_a \partial_\nu\phi\partial_\sigma\phi$$

Matrix models and gravity

define projectors on the tangential/normal bundle of $\mathcal{M} \subset \mathbb{R}^D$ as



$$\mathcal{P}_T^{ab} = g^{\mu\nu}\partial_\mu x^a \partial_\nu x^b \,,$$
$$\mathcal{P}_N^{ab} = \eta^{ab} - \mathcal{P}_T^{ab} \,,$$

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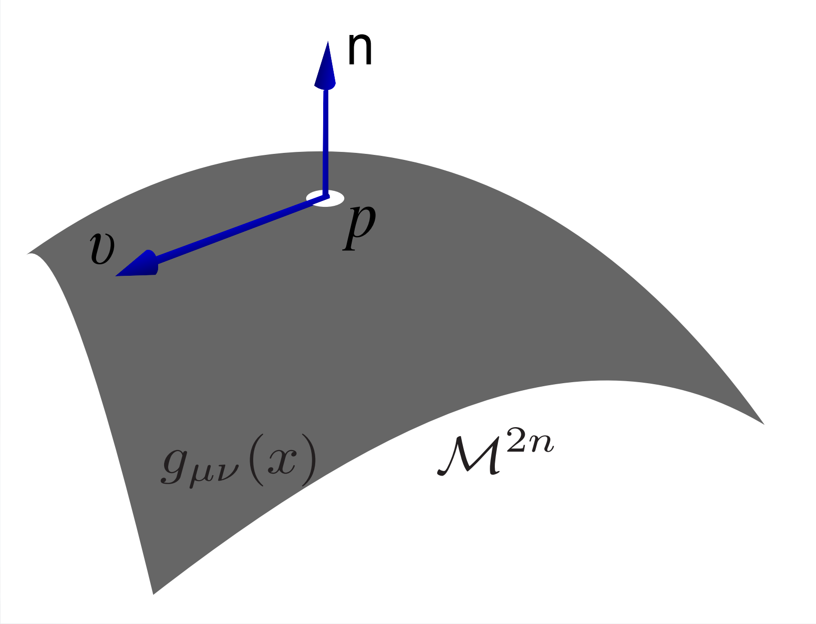

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Characteristic equation for $2n = 4$:

$$(\mathcal{J}^2)^\mu{}_\nu + \frac{(Gg)}{2}\delta^\mu{}_\nu + (\mathcal{J}^{-2})^\mu{}_\nu = 0$$

$2n = 4$: special class of geometries where $G_{\mu\nu} = g_{\mu\nu}$

i.e. $\Theta = \frac{1}{2}\theta^{-1}_{\mu\nu} dx^\mu \wedge dx^\nu \,, \qquad \star\Theta = \pm i\Theta \qquad \Rightarrow \mathcal{J}^2 = -\mathbb{1}$

NCGFT coupled to gravity

- add U(N) valued gauge fields: $X^\mu = \bar{X}^\mu + \mathcal{A}^\mu$

  $\Rightarrow \quad [X^\mu, X^\nu] \sim i(1 + \mathcal{A}^\rho \partial_\rho)\theta^{\mu\nu} + i\mathcal{F}^{\mu\nu}$

- Effective matrix model action then describes gauge fields in a gravitational background

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- However, the U(1) and SU(N) subsectors play very different roles: U(1) purely gravitational

  ⇒ non-commutative U(N) gauge field theory describes SU(N) fields coupled to gravity

  ⇒ alternative interpretation of UV/IR mixing

Introducing the IKKT model

$$S_{\rm IKKT} = {\rm Tr}\left([X^a, X^b][X_a, X_b] + \bar{\Psi}\gamma_a[X^a, \Psi]\right)$$

$$\not{D}\Psi := \gamma_a[X^a, \Psi]\,, \qquad \{\gamma_a, \gamma_b\} = 2\eta_{ab}$$

⇓

IKKT matrix model is supersymmetric and expected to be renormalizable - cf. *Nucl.Phys.* **B498** (1997) 467.

Majorana-Weyl spinor $\Psi = \mathcal{C}\bar{\Psi}^T$

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Majorana-Weyl spinor $\Psi = \mathcal{C}\bar{\Psi}^T$, is invariant under SUSY:

$$\delta^1\Psi = \frac{i}{4}[X^a, X^b][\gamma_a, \gamma_b]\epsilon\,, \qquad \delta^1 X^a = i\bar{\epsilon}\gamma^a\Psi$$

$$\delta^2\Psi = \xi\,, \qquad \delta^2 X^a = 0$$

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Further symmetries:

$$X^a \to U^{-1}X^aU\,, \qquad \Psi \to U^{-1}\Psi U\,, \qquad U \in U(\mathcal{H})\,, \quad \text{gauge inv.}$$

$$X^a \to \Lambda(g)^a_b X^b\,, \qquad \Psi_\alpha \to \tilde{\pi}(g)^\beta_\alpha\Psi_\beta\,, \qquad g \in \widetilde{SO}(D)\,, \quad \text{rotations,}$$

$$X^a \to X^a + c^a\mathbb{1}\,, \qquad c^a \in \mathbb{R}\,, \quad \text{translations}$$

IKKT model as GUT candidate?

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- Seems to provide a good candidate for quantum gravity and other fundamental interactions,

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- Originially proposed as non-perturbative definition of type IIB string theory,
- Seems to provide a good candidate for quantum gravity and other fundamental interactions,
- Here, we consider general NC brane configurations and their effective gravity in the matrix model,
- assume soft breaking of SUSY below some scale Λ and compute the effective action using a Heatkernel expansion.

The fermionic action

$$S_\Psi = \mathrm{Tr}\Psi^\dagger \not{D}\Psi = \mathrm{Tr}\Psi^\dagger \gamma_a [X^a, \Psi]$$

$$e^{-\Gamma[X]} = \int d\Psi d\Psi^\dagger e^{-S_\Psi} = (\mathrm{const.}) \exp\left(\frac{1}{2}\mathrm{Tr}\log(\not{D}^2)\right)$$

$$\not{D}^2\Psi = \gamma_a\gamma_b[X^a, [X^b, \Psi]] = (\not{D}_0^2 + V)\Psi$$

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- Consider fermions coupled to NC background
- Matrices X^a: perturbations around Moyal quantum plane

⇨ introduce NC scale $\Lambda_{NC}^4 = e^{-\sigma}$

$$[\bar{X}^\mu, \bar{X}^\nu] = i\bar{\theta}^{\mu\nu} \qquad \text{(blockdiagonal, constant)}$$

$$X^\mu = (\bar{X}^\mu + \mathcal{A}^\mu, \phi^i) = (\bar{X}^\mu - \bar{\theta}^{\mu\nu}A_\nu, \Lambda_{NC}^2\varphi^i)$$

Heatkernel expansion

$$\not{D}_0^2\Psi := \eta_{\mu\nu}[\bar{X}^\mu, [\bar{X}^\nu, \Psi]] = -\Lambda_{NC}^{-4}\bar{G}^{\mu\nu}\partial_\mu\partial_\nu\Psi$$

components of $[X^a, X^b]$:

$$[X^\mu, X^\nu] = i(\bar{\theta}^{\mu\nu} + \mathcal{F}^{\mu\nu}), \qquad [X^\mu, \phi^i] = i\bar{\theta}^{\mu\nu}D_\nu\phi^i$$

$$\mathcal{F}^{\mu\nu} = -\bar{\theta}^{\mu\rho}\bar{\theta}^{\nu\sigma}(\partial_\rho A_\sigma - \partial_\sigma A_\rho - i[A_\rho, A_\sigma]),$$

$$D_\nu\phi = \partial_\nu\phi + i[A_\nu, \phi]$$

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Consider Duhamel expansion:

$$\frac{1}{2}\mathrm{Tr}\left(\log \not{D}^2 - \log \not{D}_0^2\right) \to -\frac{1}{2}\mathrm{Tr}\int_0^\infty \frac{d\alpha}{\alpha}\left(e^{-\alpha\not{D}^2} - e^{-\alpha\not{D}_0^2}\right)e^{-\frac{\Lambda_{NC}^4}{\alpha\Lambda^2}}$$

$$= \Lambda^4\sum_{n\geq 0}\int d^4x\,\mathcal{O}\left(\frac{(p, A, \varphi)^n}{(\Lambda, \Lambda_{NC})^n}\right)$$

Small parameters of expansion

- In contrast to previous work, we consider a „semi-classical“ low energy regime characterized by

$$\epsilon(p) := p^2\Lambda^2/\Lambda_{NC}^4 \ll 1$$

- Can expand UV/IR mixing terms as

$$e^{-p^2\Lambda_{NC}^4/\alpha} \approx \sum_{m\geq 0} a_m \epsilon(p)^m$$

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- Expansion in 3 small parameters:

$$\Gamma \sim \Lambda^4 \sum_{n,l,k\geq 0} \int d^4x\, \mathcal{O}\left(\epsilon(p)^n (\frac{p^2}{\Lambda_{NC}^2})^l (\frac{p^2}{\Lambda^2})^k\right)$$

Effective NC gauge theory action

Weyl quantization map: $|p\rangle = e^{ip_\mu \bar{X}^\mu} \in \mathcal{A}$

$$\bar{P}_\mu |p\rangle = ip_\mu |p\rangle \,, \qquad \text{with } \bar{P}_\mu = -i\theta^{-1}_{\mu\nu}[\bar{X}^\nu, .]$$

$$\langle q|p\rangle = \mathrm{Tr}(|p\rangle\langle q|) = \mathrm{Tr}_{\mathcal{H}}(e^{-iq_\mu \bar{X}^\mu} e^{ip_\mu \bar{X}^\mu}) = (2\pi\Lambda_{NC}^2)^2 \delta^4(p-q)$$

$$\left[e^{ik\bar{X}}, e^{il\bar{X}}\right] = -2i\sin\left(\frac{k\bar{\theta}l}{2}\right) e^{i(k+l)\bar{X}} \,,$$

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And can now compute terms of Duhamel expansion order by order:

$$\Gamma = \frac{1}{2}\int_0^\infty d\alpha\, \mathrm{Tr}(Ve^{-\alpha \not{D}^2_0}) e^{-\frac{\Lambda^4_{NC}}{\alpha\Lambda^2}}$$

$$-\frac{1}{4}\int_0^\infty d\alpha \int_0^\alpha dt'\, \mathrm{Tr}\Big(Ve^{-t'\not{D}^2_0} Ve^{-(\alpha - t')\not{D}^2_0}\Big) e^{-\frac{\Lambda^4_{NC}}{\alpha\Lambda^2}} + \ldots$$

Gauge invar. of effective NCGFT

Adding up first 3 order contributions leads to the following order Λ^4 terms:

$$\Gamma_{\Lambda^4}(A,\varphi,p) = \frac{\Lambda^4 \mathrm{Tr}\mathbb{1}}{16\Lambda_{NC}^4}\int \frac{d^4x}{(2\pi)^2}\sqrt{g}\Big(g^{\alpha\beta}D_\alpha\varphi^i D_\beta\varphi_i$$
$$- \frac{1}{2}\Lambda_{NC}^4\big(\bar\theta^{\mu\nu}F_{\nu\mu}\bar\theta^{\rho\sigma}F_{\sigma\rho} + (\bar\theta^{\sigma\sigma'}F_{\sigma\sigma'})(F\bar\theta F\bar\theta)\big)$$
$$- 2\bar\theta^{\nu\mu}F_{\mu\alpha}g^{\alpha\beta}\partial_\nu\varphi^i\partial_\beta\varphi_i + \frac{1}{2}(\bar\theta^{\mu\nu}F_{\mu\nu})g^{\alpha\beta}\partial_\beta\varphi^i\partial_\alpha\varphi_i$$
$$+ \mathrm{h.o.}\Big)$$

⇨ **These terms are manifestly gauge invariant**

Predictive power of vacuum

Free contribution:

$$\Gamma[\bar{X}] = -\frac{1}{2}\mathrm{Tr}\int_0^\infty \frac{d\alpha}{\alpha} e^{-\alpha \not{D}_0^2 - \frac{\Lambda_{NC}^4}{\alpha\Lambda^2}} = -\frac{\Lambda^4 \mathrm{Tr}\mathbb{1}}{8}\int \frac{d^4x}{(2\pi)^2}\sqrt{g}$$

Along with general geometrical considerations, this suffices to predict loop computations!

Effective matrix model action

consider $\Gamma_L[X] = \mathrm{Tr}\mathcal{L}\,(X^a/L), \quad L = \Lambda/\Lambda^2_{NC}$

- Commutators correspond to derivative operators for gauge fields
- Leading term of eff. action can be written in terms of products of

$$J^a_b := i\Theta^{ac} g_{cb} = [X^a, X_b], \qquad \mathrm{Tr} J \equiv J^a_a = 0$$

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⇨ Most general single-trace form of effective potential + input from free contribution to the effective action:

$$\Gamma_L[X] = \mathrm{Tr}V(X) + \mathrm{h.o.},$$

$$\mathrm{Tr}V(X) = -\frac{1}{4}\mathrm{Tr}\left(\frac{L^4}{\sqrt{-\mathrm{Tr}J^4 + \frac{1}{2}(\mathrm{Tr}J^2)^2}}\right) \sim -\frac{1}{8}\int\frac{d^4x}{(2\pi)^2}\Lambda^4(x)\sqrt{g}$$

SO(D) invar. of generalized MM

Can reproduce gauge sector of induced result by a semi-classical analysis with vanishing embedding fields:

$$\frac{1}{\sqrt{\frac{1}{2}(\mathrm{Tr}J^2)^2 - \mathrm{Tr}J^4}}\Bigg|_{\partial\phi^i=0} \sim \frac{\Lambda_{NC}^4}{2}\Big(1 + \frac{1}{2}\bar{\theta}^{\mu\nu}F_{\mu\nu} + \frac{1}{4}(\bar{\theta}F)^2 + \frac{1}{4}(\bar{\theta}F)(F\bar{\theta}F\bar{\theta}) + \mathcal{O}(F^4)\Big)$$

Effective action can be written as a generalized matrix model with manifest SO(D) symmetry.

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$$+\frac{1}{4}(\bar{\theta}F)(F\bar{\theta}F\bar{\theta}) + \mathcal{O}(F^4)\Bigg)$$

Effective action can be written as a generalized matrix model with manifest SO(D) symmetry.

- Can be further generalized to include curvature terms.

$$\int \frac{d^4x}{(2\pi)^2}\sqrt{g}\,\Lambda(x)^2\Big(R + (\bar{\Lambda}_{\mathrm{NC}}^4 e^{-\sigma}\theta^{\mu\rho}\theta^{\eta\alpha}R_{\mu\rho\eta\alpha} - 4R) + c'\partial^\mu\sigma\partial_\mu\sigma\Big)$$

Bosonic action

$$S_b = -\mathrm{Tr}\left([X^a, X^b][X_a, X_b]\right)$$

- Employ background field method: $X^a \to X^a + Y^a$
- Effective action in X^a: keep only parts quadratic in Y
- Need to fix gauge for Y and add ghosts

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$$S_{gf} + S_{\text{ghost}} = -\mathrm{Tr}\left([X^a, Y_a][X^b, Y_b] - 2\bar{c}[X^a, [X_a, c]]\right)$$

⇒ leads to quadratic action:

$$S_{\text{quad}} = 2\mathrm{Tr}\left(Y^a(\Box\delta^{ab} + 2i[\Theta^{ab}, .])Y_b + 2\bar{c}\Box c\right)$$

Non-Abelian sector

background corresponding to N coinciding branes:

$$X^a = \bar{X}^a \mathbb{1}_N + \mathcal{A}^a\,, \quad \in \mathscr{A} \otimes SU(N)\,, \qquad \Theta^{ab} = \bar{\Theta}^{ab}\mathbb{1}_N + \mathcal{F}^{ab}_\alpha \lambda^\alpha$$

typical vertex term in the loop integral now looks like:

$$\begin{aligned}
&[\mathcal{F}^{ab}_\alpha(k_1)e^{ik_1X}\lambda^\alpha, f_\beta(k_2)e^{ik_2X}\lambda^\beta] \\
&= -i\sin\left(\frac{k_1\theta k_2}{2}\right)\mathcal{F}^{ab}_\alpha(k_1)f_\beta(k_2)e^{i(k_1+k_2)X}\{\lambda^\alpha,\lambda^\beta\} \\
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&\quad + \cos\left(\frac{k_1 \theta k_2}{2}\right) \mathcal{F}^{ab}_\alpha(k_1) f_\beta(k_2) e^{i(k_1+k_2)X} [\lambda^\alpha, \lambda^\beta]
\end{aligned}$$

⇒ expect low-energy effective action to reduce to $\mathcal{N} = 4$ SYM on general background $\mathcal{M}$.

SUSY breaking: $\mathcal{M} = \mathcal{M}^4 \times K$

Spontaneous symm. breaking

Consider $SU(N)$ broken down to $SU(N-1) \times U(1)$ through scalar fields $\phi^i \sim \lambda$ where λ is the generator of the unbroken $U(1)$. One-loop effective action agrees with expansion of the Dirac-Born-Infeld (DBI) action for a D3-brane in the background of $N-1$ coinciding branes

$$S_{\mathrm{DBI}} = T_3 \int_{\mathcal{M}} d^4x \frac{|\phi^2|^2}{Q} \left(\sqrt{\left| \det \left(G_{\mu\nu} + \frac{Q}{|\phi^2|^2} D_\mu \phi^i D_\nu \phi_i + \frac{Q^{1/2}}{|\phi^2|} \frac{F_{\mu\nu}}{\Lambda_{NC}^2} \right) \right|} - \sqrt{|\det G|} \right),$$

$$Q = \frac{(N-1)}{2\pi^2 \Lambda_{NC}^4}, \qquad T_3 = \Lambda_{NC}^4 \qquad \rightarrow \qquad g_s \alpha'^2 = \frac{1}{2\pi \Lambda_{NC}^4}$$

Conclusion

- Computed the effective fermion action, first from NC field theory, then from the matrix model point of view,
- SO(D) symmetry is preserved,
- Need to complete the discussion of the bosonic action and the non-Abelian sector (work in progress).

References

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2. D. N. Blaschke and H. Steinacker, On the 1-loop effective action for the IKKT model and non-commutative branes, work in progress.

3. H. Steinacker, Emergent Geometry and Gravity from Matrix Models: An Introduction, *Class.Quant. Grav.* **27** (2010) 133001, [arXiv:1012.4344].

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Thank you for your attention!