A Vector Supersymmetry Killing IR Divergences in Non-Commutative Gauge Theories

poster created by Daniel N. Blaschke,
recipient of a DOC-fellowship of the Austrian Academy of Sciences at the Institute for Theoretical Physics at Vienna University of Technology
PhD supervisor: Manfred Schweda

QFT on θ-deformed space-time

- $[x^\mu \stackrel{\star}{,} x^\nu] = \mathrm{i}\theta^{\mu\nu}$
- Weyl-Moyal $\star$-product:

$$A_\rho(x) \star A_\sigma(x) = e^{\frac{\mathrm{i}}{2}\theta^{\mu\nu}\partial^x_\mu\partial^y_\nu} A_\rho(x)A_\sigma(y)\Big|_{x=y} \neq A_\sigma(x) \star A_\rho(x)$$

- invariance under cyclic permutations of the integral

$$\int d^4x A_\mu(x) \star A_\rho(x) \star A_\sigma(x) = \int d^4x A_\sigma(x) \star A_\mu(x) \star A_\rho(x)$$

- interaction vertices gain phases, whereas propagators remain unchanged
- some Feynman integrals ("non-planar diagrams") have phases, e.g.

$$\int d^4k \frac{e^{ik\tilde{p}}}{k^2 + \mathrm{i}\epsilon} \propto \frac{1}{\tilde{p}^2} \quad \text{with} \quad \tilde{p}^\mu = \theta^{\mu\nu}p_\nu$$

- phases act as UV-regulators,

  $\Rightarrow$ origin of the *UV/IR mixing* problem

UV/IR mixing

An action such as

$$S = -\frac{1}{4}\int d^4x F_{\mu\nu} \star F^{\mu\nu} + \dots$$

with $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - \mathrm{i}g\,[A_\mu \stackrel{\star}{,} A_\nu]$
leads to (gauge independent) IR singular vacuum polarization graphs

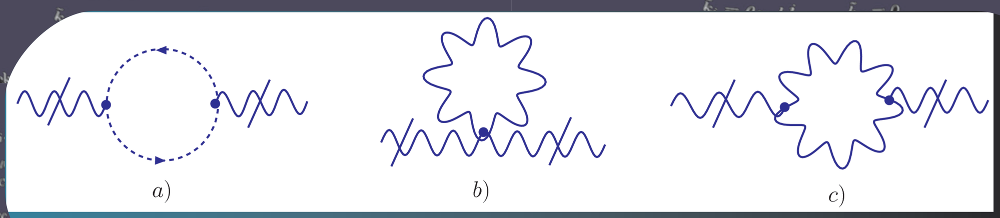
$$\Pi^{\mu\nu}_{\mathrm{IR}}(k) = \frac{2g^2}{\pi^2}\frac{\tilde{k}^\mu\tilde{k}^\nu}{(\tilde{k}^2)^2} \quad \text{with} \quad \tilde{k}^\mu = \theta^{\mu\nu}k_\nu \tag{1}$$

$\Rightarrow$ Graphs with this insertion are IR divergent!

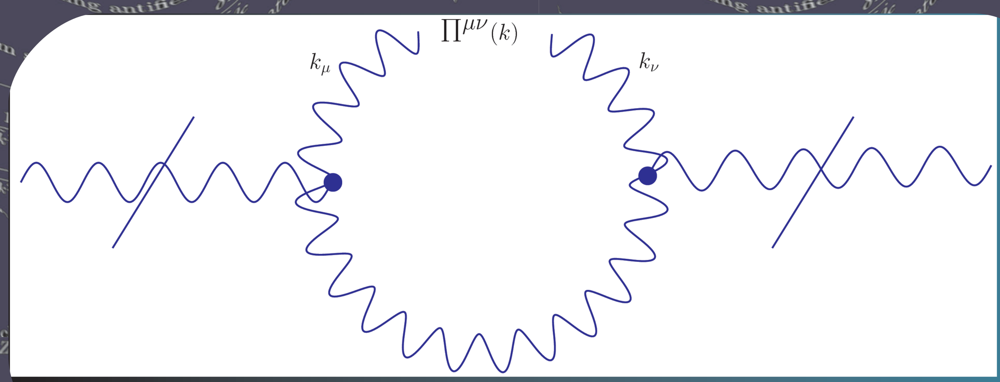
Slavnov has proposed a modification, adding to the action a term

$$\frac{1}{2}\int d^4x\, \lambda \star \theta^{\mu\nu}F_{\mu\nu} \tag{2}$$

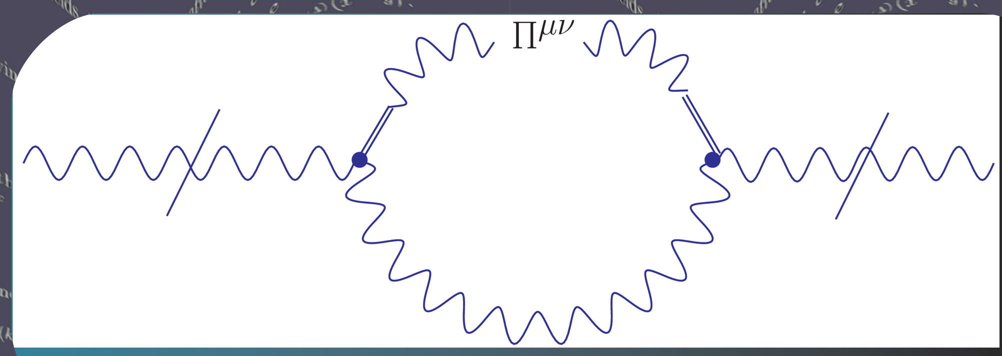
$\Rightarrow$ makes gauge field propagator transversal with respect to $\tilde{k}^\mu$



1-loop graphs of the gauge boson two-point funtion



because of the Slavnov term this graph is IR finite



an example where the Slavnov trick seems to fail

choosing an axial gauge fixing

- To avoid unitarity problems we choose the non-commutativity tensor spacelike, and
- choose an axial gauge fixing in the plane of the non-commutative coordinates:

$$\theta^{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & \theta & 0 \\ 0 & -\theta & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \qquad n^\mu = \begin{pmatrix} 0 \\ n_1 \\ n_2 \\ 0 \end{pmatrix}$$

linear vector supersymmetry

This action is invariant under the field transformations

$$\textbf{VSUSY:} \quad \begin{aligned} &\delta_i A_\mu = 0\,, & &\delta_i c = A_i\,, \\ &\delta_i \bar{c} = 0\,, & &\delta_i B = \partial_i \bar{c}\,, \\ &\delta_i \lambda = \frac{\epsilon_{ij}}{\theta} n^j \bar{c}\,, & &\delta^2 = 0\,. \end{aligned}$$

The according Ward identity yields for the gauge field propagator

$$\boxed{\Delta_{A_i A_\mu} = 0}$$

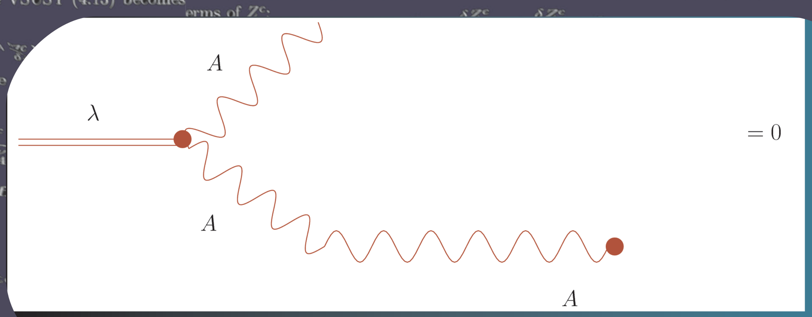
Slavnov term and BF model

- The Slavnov term, together with the gauge fixing terms, now have the form of a 2-dimensional topological BF model.
- Action:

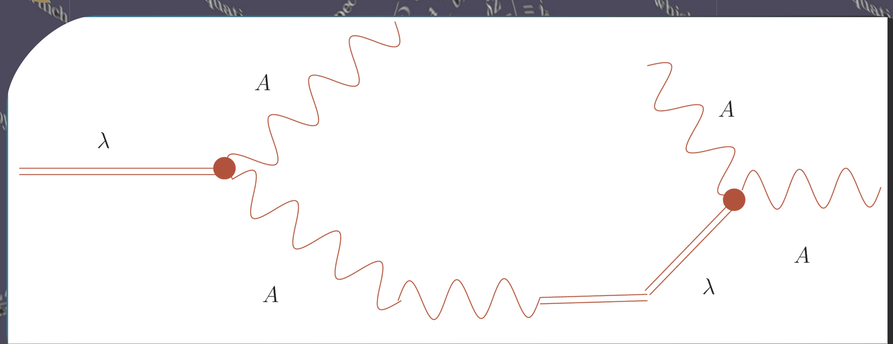
$$S = \int d^4x \left(-\frac{1}{4}F_{\mu\nu} \star F^{\mu\nu} + \frac{\theta}{2}\lambda \star \epsilon^{ij}F_{ij} + B \star n^iA_i - \bar{c} \star n^iD_ic\right)$$

where $i, j \in \{1,2\}$; $\mu,\nu \in \{0,1,2,3\}$ and

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - \mathrm{i}g\,[A_\mu \stackrel{\star}{,} A_\nu]$$
$$D_\mu c = \partial_\mu c - \mathrm{i}g\,[A_\mu \stackrel{\star}{,} c]$$



the λAA-vertex contracted with a photon propagator vanishes



building a Feynman loop graph with a λAA-vertex is impossible without a photon propagator

generalizing $\theta^{\mu\nu}$ and n^μ

- model is free of the most dangerous infrared singularities, but
- can we show cancellation of IR singular Feynman graphs for a more general choice of $\theta^{\mu\nu}$ and n^μ?

  $\Rightarrow$ The answer is yes, but we need to impose stronger Slavnov constraints:

$$S_{\mathrm{inv}} = \int d^4x \left[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\epsilon^{ijk}F_{ij}\lambda_k\right]$$

- looks like 3 dim. BF model coupled to Maxwell theory

conclusion and outlook

- Slavnov-extended Yang Mills theory can be shown to be free of worst infrared singularities, if Slavnov term is of BF-type
- More general proofs, based on a rigorous quantization scheme (perturbative or not) are desirable, though.
- Is the weaker Slavnov constraint sufficient to achieve IR finiteness?
- What are the consequences of the additional symmetries?

properties of this new action

- The action has a second gauge symmetry

$$\delta_{g2}\lambda_k = D_k\Lambda', \quad \delta_{g2}A_\mu = 0$$

- Upon choosing a space-like axial gauge fixing

$$S_{\mathrm{gf}} = \int d^4x \left[Bn^iA_i + d'n^i\lambda_i - \bar{c}n^iD_ic - \bar{\phi}n^iD_i\phi\right],$$

  where $d' = d - \mathrm{i}g\,[\bar{\phi}, c]$ is the rescaled multiplier field fixing the second gauge freedom,
- one finds invariance under the linear VSUSY

$$\begin{aligned} &\delta_i c = A_i\,, & &\delta_i\lambda_j = -\epsilon_{ijk}n^k\bar{c}\,, \\ &\delta_i B = \partial_i\bar{c}\,, & & \\ &\delta_i\Phi = 0\,, & &\text{for all other fields.} \end{aligned}$$

consequences

- same arguments as before show absence of IR singular graphs
- model exhibits numerous further symmetries including an additional symmetry of S_{inv} given by:

$$\begin{aligned} &\widehat{d}_iA_0 = -F_{i0}\,, & &\widehat{d}_i\lambda_j = \epsilon_{ijk}D_0F^{0k}\,, \\ &\widehat{d}_iA_i = 0\,. & & \end{aligned}$$

- generalization to higher dimensional models is possible, i.e. if λ had n indices the VSUSY would become

$$\begin{aligned} &\delta_i c = A_i & &\delta_i\lambda_{j_1\cdots j_n} = \epsilon_{ikj_1\cdots j_n}n^k\bar{c} \\ &\delta_i B = \partial_i\bar{c} & & \end{aligned}$$

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