Towards renormalizable models for gauge fields in non-commutative space

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$[x^\mu, x^\nu] = i\Theta^{\mu\nu}$

Background:

History

- H. Snyder (1946/47): "minimal length" to smear out point-like interactions as UV regularization in QFT
- early 1990s: J. Madore, A. Connes, T. Filk and others (fuzzy sphere, extensive study of non-commutative geometry, new IR divergences → UV/IR mixing)
- N. Seiberg & E. Witten (1999): connection to string theory (effective low-energy action on D-branes with strong B-field background, non-commutativity parameter related to B-field as $\theta \sim B^{-1}$)

Motivation

- incompatibility between GR and QFT:

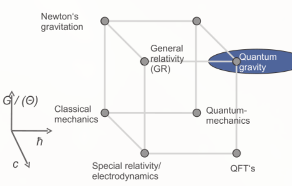
$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \langle T_{\mu\nu} \rangle \,,$$

l.h.s is the classical Einstein tensor, whereas r.h.s. is the expectation value of a quantum mechanical operator: the energy-momentum tensor $T_{\mu\nu}$

- natural limit in experimental length resolution:

$$\Delta x_\mu \simeq \text{ Planck length } \lambda_p = \sqrt{\frac{G\hbar}{c^3}} \simeq 10^{-33} cm$$

→ hint towards quantized space-time



- At high energies, a unification of all forces is expected, leading to a TOE
- The natural framework for the unified theory should be a QFT including a quantized version of GR and therefore its underlying manifold, which is space-time
- A possible way is the quantization of space-time by extension of non-commutativity from phase-space to ordinary space:

$$[\hat{x}_i, \hat{p}_j] = i\delta_{ij} \;\rightarrow\; [\hat{x}_i, \hat{x}_j] = i\theta_{ij}$$

A possible way of quantizing space-time is the introduction of non-commutative space-time coordinates

QFT on θ-deformed space-time

- consider non-commuting coordinates

$$[\hat{x}^\mu, \hat{x}^\nu] = i\theta^{\mu\nu} \qquad \Rightarrow \qquad \text{implies uncertainty relation:}$$

$$\Delta x^\mu \Delta x^\nu \geq \frac{1}{2}|\theta^{\mu\nu}| \sim (\lambda_p)^2 \,,$$

where $\lambda_p \approx 10^-33$cm is the Planck length

- definition of the Groenewold-Moyal $\star$-product:

$$f(x) \star g(x) = e^{\frac{i}{2}\theta^{\mu\nu}\partial^x_\mu \partial^y_\nu} f(x) g(y)\Big|_{x=y}$$
$$\neq g(x) \star f(x)$$

⇒ can use regular coordinates x instead of operators $\hat{x}$, since:

$$[x^\mu \stackrel{\star}{,} x^\nu] = i\theta^{\mu\nu}$$

- invariance under cyclic permutations of the integral

$$\int d^4x\, f(x) \star g(x) \star h(x) = \int d^4x\, h(x) \star f(x) \star g(x)$$
$$\Longrightarrow \int d^4x f(x) \star g(x) = \int d^4x f(x) g(x)$$

- QFT: interaction vertices gain phases, whereas propagators remain unchanged
- some Feynman integrals ("non-planar diagrams") have phases, e.g.

$$\int d^4k \frac{e^{ik\tilde{p}}}{k^2 + i\epsilon} \propto \frac{1}{\tilde{p}^2} \quad \text{with} \quad \tilde{p}^\mu = \theta^{\mu\nu} p_\nu$$

- phases act as UV-regulators,

⇒ origin of the *UV/IR mixing* problem

UV/IR mixing

"naïve" models such as

$$S_{\phi 4} = \int d^4x \left[\frac{1}{2}\left(\partial^\mu\phi \star \partial_\mu\phi + m^2 \phi \star \phi\right) + \frac{\lambda}{4!}\phi^{\star 4}\right] \,,$$
$$S_{YM} = -\frac{1}{4}\int d^4x F_{\mu\nu} \star F^{\mu\nu} + \ldots$$

with $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig\,[A_\mu \stackrel{\star}{,} A_\nu]$
lead to (gauge independent) IR singular self-energy graphs

$$\Pi^{\mu\nu}_{\phi 4, IR}(p) \propto \frac{1}{\tilde{p}^2} \,,$$
$$\Pi^{\mu\nu}_{YM, IR}(p) \propto \frac{\tilde{p}^\mu \tilde{p}^\nu}{(\tilde{p}^2)^2} \quad \text{with} \quad \tilde{p}^\mu = \theta^{\mu\nu} p_\nu$$

⇒ Graphs with these insertions are IR divergent!

Renormalizable models

- the Grosse-Wulkenhaar model (2003), where the ϕ^4 theory was supplemented by a (translation-invariance breaking) oscillator term ($\approx \tilde{x}^2\phi^2$),
- and a translation-invariant model by Gurau, Magnen, Rivasseau and Tanasa (2008), where a $\phi(-p)\frac{a^2}{\tilde{p}^2}\phi(p)$ term was added.

Gurau model

- action introduced by Gurau et. al. in arXiv:0804.0791:

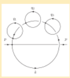
$$S = \int d^4x \left[\frac{1}{2}\left(\partial^\mu\phi \star \partial_\mu\phi + m^2\phi \star \phi - \phi \star \frac{a^2}{\tilde{\Box}}\phi\right) + \frac{\lambda}{4!}\phi^{\star 4}\right]$$

- propagator: $G(k) = \dfrac{1}{k^2 + m^2 + \frac{a^2}{\tilde{k}^2}} = \dfrac{1}{2}\sum_{\zeta=\pm 1} \dfrac{1 + \zeta\frac{m^2}{2M^2}}{k^2 + \frac{m^2}{2} + \zeta M^2}$

where $M^2 \equiv \sqrt{\frac{m^4}{4} - a^2}$

⇒ new "damping" behaviour $\lim_{k \to 0} G(k) = 0$

Damping mechanism



$$\Pi^{n\text{-pl}}_{IR}(k) \propto \frac{1}{\tilde{k}^2}$$

$$\Pi^{n \text{ npl-ins.}}(p) \equiv \lambda^2 \int d^4k \frac{e^{ik\tilde{p}}}{\left(\tilde{k}^2\right)^n \left[k^2 + m^2 + \frac{a^2}{\tilde{k}^2}\right]^{n+1}}$$

- $a = 0$: IR divergence for $n \geq 2$, i.e. integrand $\sim (k^2)^{-n}$
- $a \neq 0$: finite, i.e. integrand $\sim \frac{1}{(\tilde{k}^2)^n \left[\frac{a^2}{\tilde{k}^2}\right]^{n+1}} = \frac{\tilde{k}^2}{(a^2)^{n+1}}$

Dimensions of the Universe:

accessible by LHC $\sim 10^{-20}$m (or 10TeV)

radius of the visible universe $\sim 10^{10}$LY (or $10^{61} l_p$)

"Terra incognita"	Physically accessible

Planck length l_p $\sim 10^{-35}$m

radius of an atom $\sim 10^{-10}$m

earth's diameter $\sim 10^7$m

Gauge fields:

A non-commutative gauge field model

$$\int d^4x \phi(x) \frac{a^2}{\theta^2 \Box}\phi(x) \quad \Rightarrow \quad \int d^4x \frac{1}{4} F^{\mu\nu} \star \frac{a^2}{D^2 \widetilde{D}^2} \star F_{\mu\nu} \,,$$

where

$$U_\star(1) : \delta_\varepsilon A_\mu = \partial_\mu \varepsilon + ig\,[A_\mu \stackrel{\star}{,} \varepsilon] \,,$$
$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig\,[A_\mu \stackrel{\star}{,} A_\nu] \,,$$
$$D_\mu \bullet \equiv \partial_\mu \bullet - ig\,[A_\mu \stackrel{\star}{,} \bullet] \quad \text{and} \quad \widetilde{D}_\mu = \theta_{\mu\nu} D^\nu \,.$$

Expression $\frac{1}{D^2}F \equiv Y$ transforms covariantly ($sY = ig\,[c \stackrel{\star}{,} Y]$) and is to be understood as formal power series in the gauge field A_μ

New action

introducing auxiliary fields to avoid having infinitely many vertices:

$$S_{loc} = \int d^4x \left[\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{\lambda}{2}\left(B_{\mu\nu} + \bar{B}_{\mu\nu}\right)F^{\mu\nu} - \right.$$
$$\left. -\mu^2 \bar{B}_{\mu\nu} D^2 \widetilde{D}^2 B^{\mu\nu} + \mu^2 \bar{\psi}_{\mu\nu} D^2 \widetilde{D}^2 \psi^{\mu\nu}\right]$$

choose Landau gauge fixing:

$$S_{gf} = \int d^4x \left[b \partial_\mu A^\mu - \bar{c}\partial^\mu D_\mu c\right]$$

Gauge field propagator with IR damping:

$$G^{AA}_{\mu\nu}(k) = \frac{1}{\left(k^2 + \frac{\lambda^2}{\mu^2 \tilde{k}^2}\right)}\left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)$$

BRST invariance

The new action is almost invariant (i.e. up to a 'soft breaking') under the BRST transformations:

$$sA_\mu = D_\mu c \,, \qquad sc = igc \star c \,,$$
$$s\bar{c} = b \,, \qquad sb = 0 \,,$$
$$sF_{\mu\nu} = ig\,[c \stackrel{\star}{,} F_{\mu\nu}] \,,$$
$$s\bar{\psi}_{\mu\nu} = \bar{B}_{\mu\nu} + ig\,\{c \stackrel{\star}{,} \bar{\psi}_{\mu\nu}\} \,, \qquad s\bar{B}_{\mu\nu} = ig\,[c \stackrel{\star}{,} \bar{B}_{\mu\nu}] \,,$$
$$sB_{\mu\nu} = \psi_{\mu\nu} + ig\,[c \stackrel{\star}{,} B_{\mu\nu}] \,, \qquad s\psi_{\mu\nu} = ig\,\{c \stackrel{\star}{,} \psi_{\mu\nu}\} \,.$$

⇒ soft breaking mechanism implements the damping *without* introducing additional physical degrees of freedom (i.e. auxiliary fields are *unphysical*)!

Soft breaking

introduce additional sources to restore BRST invariance:

$$s\bar{Q}_{\mu\nu\alpha\beta} = \bar{J}_{\mu\nu\alpha\beta} + ig\,\{c \stackrel{\star}{,} \bar{Q}_{\mu\nu\alpha\beta}\} \,, \qquad s\bar{J}_{\mu\nu\alpha\beta} = ig\,[c \stackrel{\star}{,} \bar{J}_{\mu\nu\alpha\beta}] \,,$$
$$sQ_{\mu\nu\alpha\beta} = J_{\mu\nu\alpha\beta} + ig\,\{c \stackrel{\star}{,} Q_{\mu\nu\alpha\beta}\} \,, \qquad sJ_{\mu\nu\alpha\beta} = ig\,[c \stackrel{\star}{,} J_{\mu\nu\alpha\beta}] \,.$$

Soft breaking term becomes

$$S_{break} = \int d^4x \frac{\lambda}{2} B_{\mu\nu} \star F^{\mu\nu} \quad \Rightarrow \quad \int d^4x\, s\left(\bar{Q}_{\mu\nu\alpha\beta} \star B^{\mu\nu} \star F^{\alpha\beta}\right) \,.$$

"physical values":

$$\bar{Q}_{\mu\nu\alpha\beta}\big|_{phys} = 0 \,,$$
$$\bar{J}_{\mu\nu\alpha\beta}\big|_{phys} = \frac{\lambda}{4}\left(\delta_{\mu\alpha}\delta_{\nu\beta} - \delta_{\mu\beta}\delta_{\nu\alpha}\right) \,,$$

Todo

- Need to prove renormalizability of our promising gauge model candidate, however,
- standard top-down renormalization schemes fail to succeed for non-commutative theories!
- Scalar non-commutative models were proved to be renormalizable using schemes which would break gauge invariance in our model.
- Therefore, we need to develop new techniques ...

Our references

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[2] D. N. Blaschke, A. Rofner, M. Schweda and R. I. P. Sedmik, *Eur. Phys. J.* **C62** (2009) 433, [arXiv:0901.1681].

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[4] D. N. Blaschke, E. Kronberger, A. Rofner, M. Schweda, R. I. P. Sedmik and M. Wohlgenannt, [arXiv:0908.0467].

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