A Vector Supersymmetry Killing IR Divergences in Non-Commutative Gauge Theories

Talk presented by Daniel N. Blaschke

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- definition of the Weyl-Moyal $\star$-product:

$$A_\rho(x) \star A_\sigma(x) = e^{\frac{\mathrm{i}}{2}\theta^{\mu\nu}\partial^x_\mu \partial^y_\nu} A_\rho(x) A_\sigma(y)\Big|_{x=y}$$

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- invariance under cyclic permutations of the integral

$$\int d^4x A_\mu(x) \star A_\rho(x) \star A_\sigma(x) = \int d^4x A_\sigma(x) \star A_\mu(x) \star A_\rho(x)$$
$$\Longrightarrow \int d^4x A_\mu(x) \star A_\rho(x) = \int d^4x A_\mu(x) A_\rho(x)$$

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$$\int d^4k \frac{e^{ik\tilde{p}}}{k^2 + \mathrm{i}\epsilon} \propto \frac{1}{\tilde{p}^2} \quad \text{with} \quad \tilde{p}^\mu = \theta^{\mu\nu} p_\nu$$

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- phases act as UV-regulators,

  $\Rightarrow$ origin of the *UV/IR mixing* problem

UV/IR mixing in gauge theories

An action such as

$$S = -\frac{1}{4}\int d^4x F_{\mu\nu} \star F^{\mu\nu} + \dots$$

with $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - \mathrm{i}g\,[A_\mu \stackrel{\star}{,} A_\nu]$
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$$\Pi^{\mu\nu}_{\text{IR}}(k) = \frac{2g^2}{\pi^2}\frac{\tilde{k}^\mu \tilde{k}^\nu}{(\tilde{k}^2)^2} \quad \text{with} \quad \tilde{k}^\mu = \theta^{\mu\nu}k_\nu \qquad (1)$$

$\Rightarrow$ Graphs with this insertion are IR divergent!

Slavnov's extension

Slavnov has proposed a modification of Yang-Mills theories, adding to the action a term

$$\frac{1}{2}\int d^4x\,\lambda\star\theta^{\mu\nu}F_{\mu\nu} \tag{2}$$

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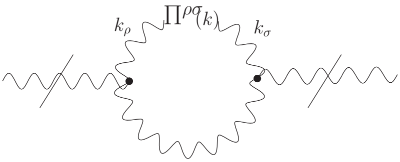

FIGURE: this graph has now become *IR finite*

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- a λ-propagator, a mixed λA-propagator and a λAA-vertex
- Slavnov trick does not work for certain diagrams, i.e.

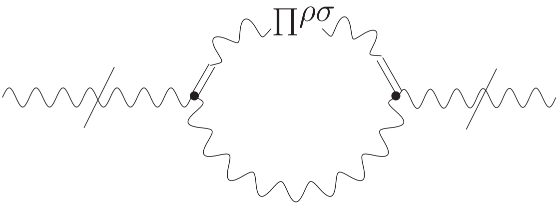

FIGURE: this graph is *IR divergent*

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- The Slavnov term, together with the gauge fixing terms, now have the form of a 2-dimensional topological BF model.

- Action:

$$S_{\text{inv}} = \int d^4x \, (-\frac{1}{4} F_{\mu\nu} \star F^{\mu\nu} + \frac{\theta}{2} \lambda \star \epsilon^{ij} F_{ij})$$

where

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- and

$$S_{\rm gf} = \int d^4x \; (B \star n^i A_i - \bar{c} \star n^i D_i c)$$

with

$$D_\mu c = \partial_\mu c - \mathrm{i}g \left[A_\mu \stackrel{\star}{,} c\right]$$

Symmetries of the Action

S is invariant under

BRS:

$$
\begin{aligned}
sA_\mu &= D_\mu c\,, & s\bar{c} &= B\,,\\
s\lambda &= -\mathrm{i}g\,[\lambda, c]\,, & sB &= 0\,,\\
sc &= \frac{\mathrm{i}g}{2}\,[c, c]\,, & s^2 &= 0\,.
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VSUSY:

$$\delta_i A_\mu = 0\,, \qquad \delta_i c = A_i\,,$$
$$\delta_i \bar{c} = 0\,, \qquad \delta_i B = \partial_i \bar{c}\,,$$
$$\delta_i \lambda = \frac{\epsilon_{ij}}{\theta} n^j \bar{c}\,, \qquad \delta^2 = 0\,.$$

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Note: Only the interplay of appropriate choices for $\theta^{\mu\nu}$ and n^{μ} lead to the existence of the VSUSY.

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In contrast to the pure topological theories, we have an additional vectorial symmetry:

$$\widehat{d}_i A_J = -F_{iJ}\,, \qquad \widehat{d}_i \lambda = -\frac{\epsilon_{ij}}{\theta} D_K F^{Kj}\,,$$
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$\Rightarrow$ The algebra involving s, δ_i, $\widehat{d}_i$ and the $(1,2)$-plane translation generator ∂_i closes on-shell.

Legendre transformation

$$S_{\text{tot}}[\phi, \phi^*, ...] \to Z^c[j_\phi, \phi^*, ...]$$

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$\Rightarrow$ WI for VSUSY at vanishing antifields transforms into

$$\mathcal{W}_i Z^c \Big|_{\{A^*,\lambda^*,c^*\}\to 0} = 0$$

$$\Rightarrow \int d^4x \left\{ j_B\, \partial_i \frac{\delta Z^c}{\delta j_{\bar{c}}} - j_c \frac{\delta Z^c}{\delta j_A^i} + \frac{\epsilon_{ij}}{\theta} n^j j_\lambda \frac{\delta Z^c}{\delta j_{\bar{c}}} \right\} = 0$$

… and its consequences

Differentiating this with respect to j_c and j_A^μ yields for the gauge field propagator:

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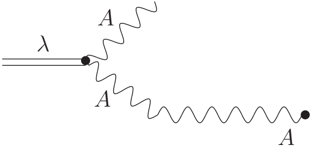

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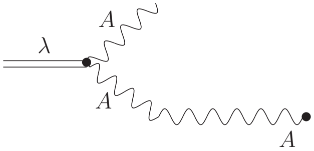


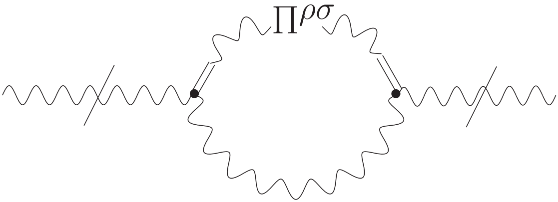
$= 0$

- It is impossible to construct a closed loop including a λAA-vertex without having such a combination somewhere.

$\Longrightarrow$ *All loop graphs involving the λAA-vertex vanish!*

Absence of IR singularities

In particular, dangerous vacuum polarization insertions as in the following figure vanish:



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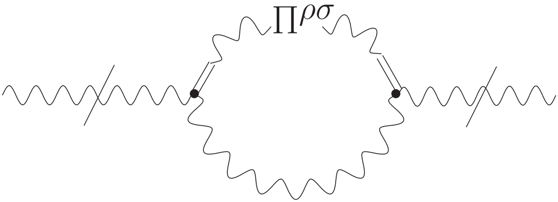

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$\Rightarrow$ model is free of the most dangerous infrared singularities!

More general choice of $\theta^{\mu\nu}$ and n^μ

- Can we show cancellation of IR singular Feynman graphs for a more general choice of $\theta^{\mu\nu}$ and n^μ?

jump to end

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More general choice of $\theta^{\mu\nu}$ and n^μ

- Can we show cancellation of IR singular Feynman graphs for a more general choice of $\theta^{\mu\nu}$ and n^μ?

  $\Rightarrow$ The answer is yes, but we need to impose stronger Slavnov constraints
- initial Slavnov constraint was $\theta^{12}F_{12} + \theta^{13}F_{13} + \theta^{23}F_{23} = 0$
- Upon introducing stronger constraints we may write

$$S_{\text{inv}} = \int d^4x \left[-\frac{1}{4} F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\epsilon^{ijk}F_{ij}\lambda_k \right]$$

  with $i, j, k \in \{1, 2, 3\}$
- Looks like 3 dim. BF model coupled to Maxwell theory

jump to end

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<u>Difference:</u> This symmetry is broken when fixing the second gauge symmetry!

The vector supersymmetry

The gauge fixed action, with space-like axial gauge

$$S_{\mathrm{gf}} = \int d^4x \left[B n^i A_i + d' n^i \lambda_i - \bar{c} n^i D_i c - \bar{\phi} n^i D_i \phi \right] ,$$

is invariant under the linear VSUSY

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($d' = d - \mathrm{i}g\left[\bar{\phi}, c\right]$ is the rescaled multiplier field fixing the second gauge freedom.)

The vector supersymmetry

Ward identity describing the linear vector supersymmetry in terms of Z^c is given by

$$\mathcal{W}_i Z^c = \int d^4x \left[j_B \partial_i \frac{\delta Z^c}{\delta j_{\bar{c}}} - j_c \frac{\delta Z^c}{\delta j_A^i} + \epsilon_{ijk} n^j j_\lambda^k \frac{\delta Z^c}{\delta j_{\bar{c}}} \right] = 0.$$

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- same arguments as before show absence of IR singular graphs
- model exhibits numerous further symmetries
- generalization to higher dimensional models is possible, i.e. if λ had n indices the VSUSY would become

$$\delta_i c = A_i \qquad \delta_i \lambda_{j_1 \cdots j_n} = \epsilon_{ikj_1 \cdots j_n} n^k \bar{c}$$
$$\delta_i B = \partial_i \bar{c}$$

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- What is the role of VSUSY with respect to UV/IR mixing in topological NCGFT in general?

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- SUSY, in the form of VSUSY, seems to play a decisive role in theories which are not Poincaré supersymmetric.
- What is the role of VSUSY with respect to UV/IR mixing in topological NCGFT in general?
- What are the consequences of the additional symmetries?

VSUSY Killing IR Divergences in NCGFT

Talk presented by Daniel N. Blaschke

References

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Thank you for your attention!