Heat Kernel Expansion and Induced Action for Matrix Models

Talk presented by Daniel N. Blaschke

Recipient of an APART -fellowship of the Austrian Academy of Sciences at the

University of Vienna,
Faculty of Physics,
Mathematical Physics Group

Collaborators: H. Steinacker, M. Wohlgenannt

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Outline

i. Introduction:

- YM matrix models
- IKKT Model

ii. Induced fermion action

- as NCGFT
- as generalized matrix model

Matrix models of Yang-Mills type

$$S_{YM} = -\mathrm{Tr}[X^a, X^b][X^c, X^d]\eta_{ac}\eta_{bd}$$

X^a Herm. matrices on $\mathcal{H}$, and η_{ab} is D-dim. flat metric

$$X^a = \left(X^\mu, \Phi^i\right), \ \mu = 1, \ldots, 2n, \ i = 1, \ldots, D - 2n,$$

so that $\Phi^i(X) \sim \phi^i(x)$ define embedding $\mathcal{M}^{2n} \hookrightarrow \mathbf{R}^D$

$g_{\mu\nu}(x) = \partial_\mu x^a \partial_\nu x^b \eta_{ab}$ (in semi-classical limit)

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$\mathcal{M}^{2n}$ endowed with a Poisson structure

$-i[X^\mu, X^\nu] \sim \{x^\mu, x^\nu\}_{pb} = \theta^{\mu\nu}(x) \Rightarrow$ "effective" metric

$$G^{\mu\nu} = e^{-\sigma}\theta^{\mu\rho}\theta^{\nu\sigma}g_{\rho\sigma} = -(\mathcal{J}^2)^\mu_\rho g^{\rho\nu}, \qquad e^{-\sigma} \equiv \frac{\sqrt{\det\theta^{-1}_{\mu\nu}}}{\sqrt{\det G_{\rho\sigma}}}$$

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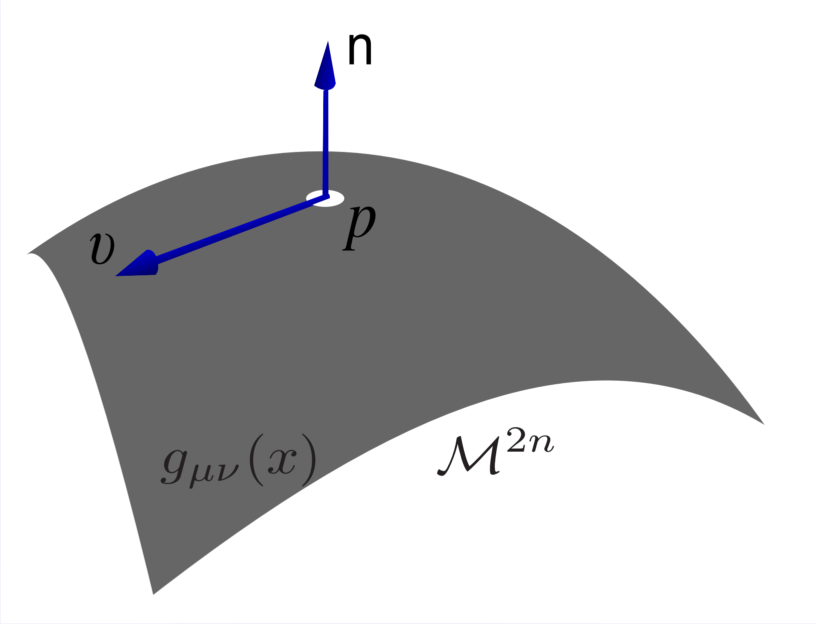
$$-\mathrm{Tr}[X^a, \Phi][X_a, \Phi] \sim \int d^4x \sqrt{\det\theta^{-1}}\,\theta^{\mu\nu}\theta^{\rho\sigma}\partial_\mu x^a \partial_\rho x_a \partial_\nu\phi\partial_\sigma\phi$$

Matrix models and gravity

define projectors on the tangential/normal bundle of $\mathcal{M} \subset \mathbb{R}^D$ as



$$\mathcal{P}_T^{ab} = g^{\mu\nu}\partial_\mu x^a \partial_\nu x^b\,,$$
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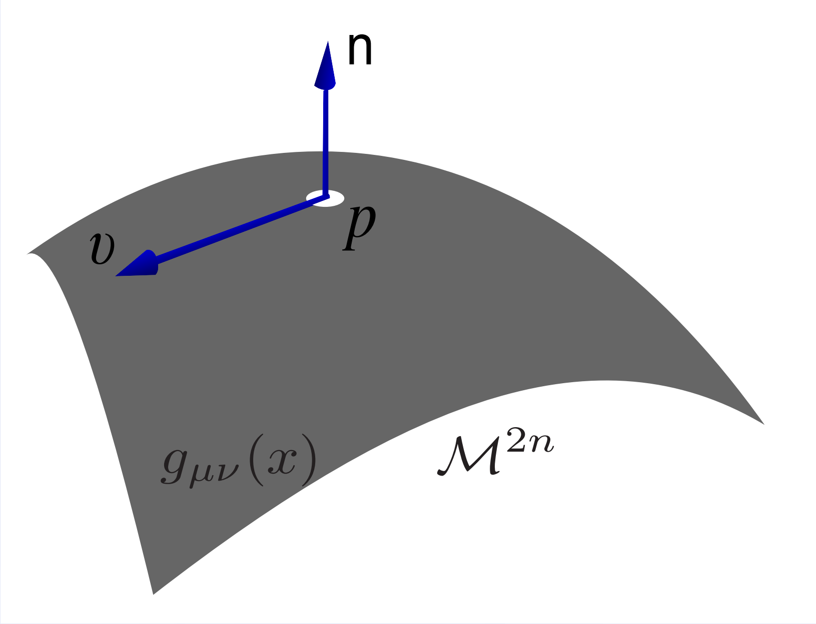

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Characteristic equation for $2n = 4$:

$$(\mathcal{J}^2)^\mu{}_\nu + \frac{(Gg)}{2}\delta^\mu{}_\nu + (\mathcal{J}^{-2})^\mu{}_\nu = 0$$

$2n = 4$: special class of geometries where $G_{\mu\nu} = g_{\mu\nu}$

i.e. $\Theta = \frac{1}{2}\theta^{-1}_{\mu\nu} dx^\mu \wedge dx^\nu \,, \qquad \star\Theta = \pm i\Theta \qquad \Rightarrow \mathcal{J}^2 = -\mathbb{1}$

NCGFT coupled to gravity

- add U(N) valued gauge fields: $X^\mu = \bar{X}^\mu + \mathcal{A}^\mu$

  $\Rightarrow \quad [X^\mu, X^\nu] \sim i(1 + \mathcal{A}^\rho \partial_\rho)\theta^{\mu\nu} + i\mathcal{F}^{\mu\nu}$

- Effective matrix model action then describes gauge fields in a gravitational background

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- However, the U(1) and SU(N) subsectors play very different roles: U(1) purely gravitational

  ⇒ non-commutative U(N) gauge field theory describes SU(N) fields coupled to gravity

  ⇒ alternative interpretation of UV/IR mixing

Introducing the IKKT model

$$S_{\mathrm{IKKT}} = \mathrm{Tr}\left([X^a, X^b][X_a, X_b] + \bar{\Psi}\gamma_a[X^a, \Psi]\right)$$

$$\not{D}\Psi := \gamma_a[X^a, \Psi]\,, \qquad \{\gamma_a, \gamma_b\} = 2\eta_{ab}$$

IKKT matrix model is supersymmetric and expected to be renormalizable - cf. *Nucl.Phys.* **B498** (1997) 467.

Majorana-Weyl spinor $\Psi = \mathcal{C}\bar{\Psi}^T$

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Majorana-Weyl spinor $\Psi = \mathcal{C}\bar{\Psi}^T$, is invariant under SUSY:

$$\delta^1\Psi = \frac{i}{4}[X^a, X^b][\gamma_a, \gamma_b]\epsilon\,, \qquad \delta^1 X^a = i\bar{\epsilon}\gamma^a\Psi$$

$$\delta^2\Psi = \xi\,, \qquad \delta^2 X^a = 0$$

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Further symmetries:

$X^a \to U^{-1}X^aU\,,$	$\Psi \to U^{-1}\Psi U\,,$	$U \in U(\mathcal{H})\,,$	gauge inv.
$X^a \to \Lambda(g)^a_b X^b\,,$	$\Psi_\alpha \to \tilde{\pi}(g)^\beta_\alpha \Psi_\beta\,,$	$g \in \widetilde{SO}(D)\,,$	rotations,
$X^a \to X^a + c^a\mathbb{1}\,,$		$c^a \in \mathbb{R}\,,$	translations

IKKT model as TOE?

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- Seems to provide a good candidate for quantum gravity and other fundamental interactions,

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- Originially proposed as non-perturbative definition of type IIB string theory,
- Seems to provide a good candidate for quantum gravity and other fundamental interactions,
- Here, we consider general NC brane configurations and their effective gravity in the matrix model,
- assume soft breaking of SUSY below some scale Λ and compute the effective action using a Heatkernel expansion.

The fermionic action

$$S_\Psi = \mathrm{Tr}\Psi^\dagger \not{D}\Psi = \mathrm{Tr}\Psi^\dagger \gamma_a [X^a, \Psi]$$

$$e^{-\Gamma[X]} = \int d\Psi d\Psi^\dagger e^{-S_\Psi} = (\mathrm{const.}) \exp\left(\frac{1}{2}\mathrm{Tr}\log(\not{D}^2)\right)$$

$$\not{D}^2\Psi = \gamma_a\gamma_b[X^a, [X^b, \Psi]] = (\not{D}_0^2 + V)\Psi$$

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- Consider fermions coupled to NC background
- Matrices X^a: perturbations around Moyal quantum plane

introduce NC scale $\Lambda_{NC}^4 = e^{-\sigma}$

$$[\bar{X}^\mu, \bar{X}^\nu] = i\bar{\theta}^{\mu\nu} \qquad \text{(blockdiagonal, constant)}$$

$$X^\mu = (\bar{X}^\mu + \mathcal{A}^\mu, \phi^i) = (\bar{X}^\mu - \bar{\theta}^{\mu\nu}A_\nu, \Lambda_{NC}^2\varphi^i)$$

Heatkernel expansion

$$\not{D}_0^2\Psi := \eta_{\mu\nu}[\bar{X}^\mu, [\bar{X}^\nu, \Psi]] = -\Lambda_{NC}^{-4}\bar{G}^{\mu\nu}\partial_\mu\partial_\nu\Psi$$

components of $[X^a, X^b]$:

$$[X^\mu, X^\nu] = i(\bar{\theta}^{\mu\nu} + \mathcal{F}^{\mu\nu}), \qquad [X^\mu, \phi^i] = i\bar{\theta}^{\mu\nu}D_\nu\phi^i$$

$$\mathcal{F}^{\mu\nu} = -\theta^{\mu\rho}\theta^{\nu\sigma}(\partial_\rho A_\sigma - \partial_\sigma A_\rho - i[A_\rho, A_\sigma]),$$

$$D_\nu\phi = \partial_\nu\phi + i[A_\nu, \phi]$$

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Consider Duhamel expansion:

$$\frac{1}{2}\mathrm{Tr}\left(\log\not{D}^2 - \log\not{D}_0^2\right) \to -\frac{1}{2}\mathrm{Tr}\int_0^\infty \frac{d\alpha}{\alpha}\left(e^{-\alpha\not{D}^2} - e^{-\alpha\not{D}_0^2}\right)e^{-\frac{\Lambda_{NC}^4}{\alpha\Lambda^2}}$$

$$= \Lambda^4\sum_{n\geq 0}\int d^4x\,\mathcal{O}\left(\frac{(p, A, \varphi)^n}{(\Lambda, \Lambda_{NC})^n}\right)$$

Small parameters of expansion

- In contrast to previous work, we consider a „semi-classical“ low energy regime characterized by

$$\epsilon(p) := p^2\Lambda^2/\Lambda_{NC}^4 \ll 1$$

- Can expand UV/IR mixing terms as

$$e^{-p^2\Lambda_{NC}^4/\alpha} \approx \sum_{m\geq 0} a_m \epsilon(p)^m$$

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- Expansion in 3 small parameters:

$$\Gamma \sim \Lambda^4 \sum_{n,l,k\geq 0} \int d^4x\, \mathcal{O}\left(\epsilon(p)^n (\frac{p^2}{\Lambda_{NC}^2})^l (\frac{p^2}{\Lambda^2})^k\right)$$

Specifying the Hilbert space

inner product: $\langle\Psi_1, \Psi_2\rangle = \mathrm{Tr}_{\mathcal{H}} \Psi_1^\dagger \Psi_2 = \Lambda_{NC}^4 \int \frac{d^4x}{(2\pi)^2} \sqrt{g} \Psi_1^\dagger \Psi_2$

Weyl quantization map: $|p\rangle = e^{ip_\mu \bar{X}^\mu} \in \mathcal{A}$

$\bar{P}_\mu |p\rangle = ip_\mu |p\rangle \,, \qquad \text{with } \bar{P}_\mu = -i\theta^{-1}_{\mu\nu}[\bar{X}^\nu, .]$

$\langle q|p\rangle = \mathrm{Tr}(|p\rangle\langle q|) = \mathrm{Tr}_{\mathcal{H}}(e^{-iq_\mu \bar{X}^\mu} e^{ip_\mu \bar{X}^\mu}) = (2\pi\Lambda_{NC}^2)^2 \delta^4(p-q)$

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$$|\Psi\rangle = \int \frac{d^4p}{(2\pi\Lambda_{NC}^2)^2} |p\rangle\langle p|\Psi\rangle = \int \frac{d^4p}{(2\pi\Lambda_{NC}^2)^2} \psi(p)\, e^{ip_\mu \bar{X}^\mu}$$

$$\left[e^{ik\bar{X}}, e^{il\bar{X}}\right] = -2i \sin\left(\frac{k\bar{\theta}l}{2}\right) e^{i(k+l)\bar{X}} \,,$$

$$\not{D}_0^2 |p\rangle = \Lambda_{NC}^{-4} \bar{G}^{\mu\nu} p_\mu p_\nu |p\rangle$$

Effective NC gauge theory action

general matrix element: $\langle \Psi'_\beta | V | \Psi_\alpha \rangle$

And can now compute terms of Duhamel expansion order by order:

$$\Gamma = \frac{1}{2} \int_0^\infty d\alpha \, \mathrm{Tr}(V e^{-\alpha \not{D}_0^2}) e^{-\frac{\Lambda_{NC}^4}{\alpha \Lambda^2}}$$

$$- \frac{1}{4} \int_0^\infty d\alpha \int_0^\alpha dt' \; \mathrm{Tr}\Big(V e^{-t' \not{D}_0^2} V e^{-(\alpha - t') \not{D}_0^2}\Big) e^{-\frac{\Lambda_{NC}^4}{\alpha \Lambda^2}} + \dots$$

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E.g. first order:

$$\Gamma^1 = \frac{\Lambda^4 \mathrm{Tr} \mathbb{1}}{16 \Lambda_{NC}^8} \int \frac{d^4 l}{(2\pi \Lambda_{NC}^2)^2} \sqrt{g} \, l^2 \big(\bar{G}^{\mu\nu} A_\mu(-l) A_\nu(l) + \varphi^i(-l) \varphi_i(l) \big)$$
$$+ \mathcal{O}(l^4)$$

Gauge invar. of effective NCGFT

Together with 2nd and 3rd order contributions, that leads to order Λ^4 terms:

$$\begin{aligned}\Gamma_{\Lambda^4}(A,\varphi,p) = \frac{\Lambda^4 \mathrm{Tr}\mathbb{1}}{16\Lambda_{NC}^4}\int \frac{d^4x}{(2\pi)^2}\sqrt{g}\Big(& g^{\alpha\beta}D_\alpha\varphi^i D_\beta\varphi_i \\ & - \frac{1}{2}\Lambda_{NC}^4\big(\bar\theta^{\mu\nu}F_{\nu\mu}\bar\theta^{\rho\sigma}F_{\sigma\rho} + (\bar\theta^{\sigma\sigma'}F_{\sigma\sigma'})(F\bar\theta F\bar\theta)\big) \\ & - 2\bar\theta^{\nu\mu}F_{\mu\alpha}g^{\alpha\beta}\partial_\nu\varphi^i\partial_\beta\varphi_i + \frac{1}{2}(\bar\theta^{\mu\nu}F_{\mu\nu})g^{\alpha\beta}\partial_\beta\varphi^i\partial_\alpha\varphi_i \\ & + \mathrm{h.o.}\Big)\end{aligned}$$

➡ These terms are manifestly gauge invariant

Predictive power of vacuum

Free contribution:

$$\Gamma[\bar{X}] = -\frac{1}{2}\mathrm{Tr}\int_0^\infty \frac{d\alpha}{\alpha} e^{-\alpha \not{D}_0^2 - \frac{\Lambda_{NC}^4}{\alpha\Lambda^2}} = -\frac{\Lambda^4 \mathrm{Tr}\mathbb{1}}{8}\int \frac{d^4x}{(2\pi)^2}\sqrt{g}$$

Along with general geometrical considerations, this suffices to predict loop computations!

Effective matrix model action

$$\text{consider } \Gamma_L[X] = \mathrm{Tr}\mathcal{L}\,(X^a/L), \qquad L = \Lambda/\Lambda_{NC}^2$$

- Commutators correspond to derivative operators for gauge fields
- Leading term of effective action can be written in terms of products of

$$J^a_b := i\Theta^{ac} g_{cb} = [X^a, X_b], \qquad \mathrm{Tr}J \equiv J^a_a = 0$$

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$$J_b^a := i\Theta^{ac}g_{cb} \; = \; [X^a, X_b], \qquad \mathrm{Tr}J \equiv J_a^a = 0$$

- Recall semi-classical characteristic equation

$$(J^4)_b^a - \frac{1}{2}(\mathrm{Tr}J^2)\,(J^2)_b^a \; \sim \; -\Lambda_{NC}^{-8}(x)(\mathcal{P}_T)_b^a\,,$$

$$J^5 - \frac{1}{2}(\mathrm{Tr}J^2)\; J^3 \; \sim \; -\Lambda_{NC}^{-8}(x)\,J\,, \qquad \mathrm{Tr}J^2 \sim \Lambda_{NC}^{-4}(x)(Gg)\,,$$

$\mathcal{P}_T^{ab} := g^{\mu\nu}\partial_\mu x^a \partial_\nu x^b$ ⇨ Projector on tangential bundle

Generalized matrix model

- Most general single-trace form of effective potential:

$$V(X) = V\Big(-\frac{L^4}{\mathrm{Tr}J^2}, \frac{-\mathrm{Tr}J^4 + \frac{1}{2}(\mathrm{Tr}J^2)^2}{(\mathrm{Tr}J^2)^2}\Big)$$

$$\sim V\Big(\frac{L^4}{\Lambda_{NC}^{-4}(x)\,(Gg)}, \frac{4}{(Gg)^2}\Big)$$

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$$\sim V\Big(\frac{L^4}{\Lambda_{NC}^{-4}(x)\,(Gg)}, \frac{4}{(Gg)^2}\Big)$$

The exact form can be determined from the free contribution to the effective action introduced previously:

$$\Gamma_L[X] = \mathrm{Tr}V(X) + \mathrm{h.o.},$$

$$\mathrm{Tr}V(X) = -\frac{1}{4}\mathrm{Tr}\Big(\frac{L^4}{\sqrt{-\mathrm{Tr}J^4 + \frac{1}{2}(\mathrm{Tr}J^2)^2}}\Big) \sim -\frac{1}{8}\int\frac{d^4x}{(2\pi)^2}\Lambda^4(x)\sqrt{g}$$

SO(D) invar. of generalized MM

Can now reproduce e.g. the gauge sector of the induced result displayed previously, by a semi-classical analysis with vanishing embedding fields:

$$\frac{1}{\sqrt{\frac{1}{2}(\mathrm{Tr}J^2)^2 - \mathrm{Tr}J^4}}\Bigg|_{\partial\phi^i=0} \sim \frac{\Lambda_{NC}^4}{2}\Bigg(1 + \frac{1}{2}\bar{\theta}^{\mu\nu}F_{\mu\nu} + \frac{1}{4}(\bar{\theta}F)^2 + \frac{1}{4}(\bar{\theta}F)(F\bar{\theta}F\bar{\theta}) + \mathcal{O}(F^4)\Bigg)$$

Effective action can be written as a generalized matrix model with manifest SO(D) symmetry.

Generalized MM & curvature

Generalizing the effective matrix model action to include curvature terms porportional to Λ^2:

$$\Gamma_L[X] = -\frac{1}{4}\mathrm{Tr}\left(\frac{L^4}{\sqrt{-\mathrm{Tr}J^4 + \frac{1}{2}(\mathrm{Tr}J^2)^2 + \frac{1}{L^2}\mathcal{L}_{10,\,\mathrm{curv}}[X] + \dots}}\right)$$

$$\sim -\int \frac{d^4x\sqrt{G}}{8(2\pi)^2}\left(\Lambda^4(x) - \frac{1}{8}\Lambda^2(x)\Lambda_{NC}^{12}(x)\mathcal{L}_{10,\,\mathrm{curv}} + \dots\right)$$

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$$\text{example for} \quad G_{\mu\nu} = g_{\mu\nu}:$$

$$\mathrm{Tr}\,\Lambda_{\mathrm{NC}}^{12}[X]\mathcal{L}_{10,\mathrm{c}} \sim \int \frac{d^4x}{(2\pi)^2}\sqrt{g}\,\Lambda(x)^2\Big(R + (\bar{\Lambda}_{\mathrm{NC}}^4 e^{-\sigma}\theta^{\mu\rho}\theta^{\eta\alpha}R_{\mu\rho\eta\alpha} - 4R) + c'\partial^\mu\sigma\partial_\mu\sigma\Big)$$

Analogs of Seeley-de Witt coefficients corresponding to induced gravity.

Outlook: bosonic action

$$S_b = -\mathrm{Tr}\left([X^a, X^b][X_a, X_b]\right)$$

- Employ background field method: $X^a \to X^a + Y^a$
- Effective action in X^a: keep only parts quadratic in Y
- Need to fix gauge for Y and add ghosts

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$$S_{gf} + S_{\text{ghost}} = -\mathrm{Tr}\left([X^a, Y_a][X^b, Y_b] - 2\bar{c}[X^a, [X_a, c]]\right)$$

⇒ leads to quadratic action:

$$S_{\text{quad}} = 2\mathrm{Tr}\left(Y^a(\Box\delta^{ab} + 2i[\Theta^{ab}, .])Y_b + 2\bar{c}\Box c\right)$$

Conclusion

- Computed the effective fermion action, first from NC field theory, then from the matrix model point of view,
- SO(D) symmetry is preserved,
- Need to discuss the bosonic action (work in progress).

References

1. D. N. Blaschke, H. Steinacker and M. Wohlgenannt, Heat kernel expansion and induced action for the matrix model Dirac operator, *JHEP* **03** (2011) 002, [arXiv:1012.4344].

2. D. N. Blaschke and H. Steinacker, On the 1-loop effective action for matrix models, non-commutative branes and SUSY breaking, work in progress.

3. H. Steinacker, Emergent Geometry and Gravity from Matrix Models: An Introduction, *Class.Quant. Grav.* **27** (2010) 133001, [arXiv:1012.4344].

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1. D. N. Blaschke, H. Steinacker and M. Wohlgenannt, Heat kernel expansion and induced action for the matrix model Dirac operator, *JHEP* **03** (2011) 002, [arXiv:1012.4344].

2. D. N. Blaschke and H. Steinacker, On the 1-loop effective action for matrix models, non-commutative branes and SUSY breaking, work in progress.

3. H. Steinacker, Emergent Geometry and Gravity from Matrix Models: An Introduction, *Class.Quant. Grav.* **27** (2010) 133001, [arXiv:1012.4344].

Thank you for your attention!