# Translation-invariant models for non-commutative gauge fields

Talk presented by Daniel N. Blaschke

Institute for Theoretical Physics,
Vienna University of Technology

*Collaborators:* F. Gieres, H. Grosse, E. Kronberger, M. Schweda, R. Sedmik, M. Wohlgenannt

May 17, 2008

# Weyl-Moyal correspondence

- assume non-commuting space-time coordinates:

$$[\hat{x}^\mu, \hat{x}^\nu] = \mathrm{i}\theta^{\mu\nu}\,, \qquad \Rightarrow \text{leads to uncertainty relation:}$$

$$\Delta x^\mu \Delta x^\nu \geq \frac{1}{2}|\theta^{\mu\nu}| \sim (\lambda_p)^2$$

# Weyl-Moyal correspondence



- assume non-commuting space-time coordinates:

$$[\hat{x}^\mu, \hat{x}^\nu] = \mathrm{i}\theta^{\mu\nu}\,, \qquad \Rightarrow \text{ leads to uncertainty relation:}$$

$$\Delta x^\mu \Delta x^\nu \geq \frac{1}{2}|\theta^{\mu\nu}| \sim (\lambda_p)^2$$

- definition of the Groenewold-Moyal $\star$-product:

$$f(x) \star g(x) = e^{\frac{\mathrm{i}}{2}\theta^{\mu\nu}\partial^x_\mu \partial^y_\nu} f(x) g(y)\Big|_{x=y}$$

$$\neq g(x) \star f(x)$$

# Weyl-Moyal correspondence

- assume non-commuting space-time coordinates:

$$[\hat{x}^\mu, \hat{x}^\nu] = \mathrm{i}\theta^{\mu\nu}\,, \qquad \Rightarrow \text{leads to uncertainty relation:}$$

$$\Delta x^\mu \Delta x^\nu \geq \frac{1}{2}|\theta^{\mu\nu}| \sim (\lambda_p)^2$$

- definition of the Groenewold-Moyal $\star$-product:

$$f(x) \star g(x) = e^{\frac{\mathrm{i}}{2}\theta^{\mu\nu}\partial^x_\mu \partial^y_\nu} f(x) g(y)\Big|_{x=y}$$

$$\neq g(x) \star f(x)$$

- invariance under cyclic permutations of the integral

$$\int d^4x f(x) \star g(x) \star h(x) = \int d^4x h(x) \star f(x) \star g(x)$$

$$\Longrightarrow \int d^4x f(x) \star g(x) = \int d^4x f(x) g(x)$$

# QFT on $\theta$-deformed space-time

For a field theory this means:

- interaction vertices gain phases, whereas propagators remain unchanged

# QFT on $\theta$-deformed space-time

For a field theory this means:

- interaction vertices gain phases, whereas propagators remain unchanged
- some Feynman integrals ("non-planar diagrams") have phases, e.g.

$$\int d^4k \frac{e^{\mathrm{i}k\tilde{p}}}{k^2 + \mathrm{i}\epsilon} \propto \frac{1}{\tilde{p}^2} \quad \text{with} \quad \tilde{p}^\mu = \theta^{\mu\nu} p_\nu$$

# QFT on $\theta$-deformed space-time



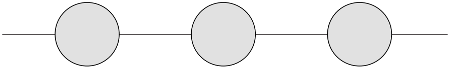
For a field theory this means:

- interaction vertices gain phases, whereas propagators remain unchanged
- some Feynman integrals ("non-planar diagrams") have phases, e.g.

$$\int d^4k \frac{e^{\mathrm{i}k\tilde{p}}}{k^2 + \mathrm{i}\epsilon} \propto \frac{1}{\tilde{p}^2} \quad \text{with} \quad \tilde{p}^\mu = \theta^{\mu\nu} p_\nu$$

- phases act as UV-regulators,

  $\Rightarrow$ origin of the *UV/IR mixing* problem



# RECENT SUCCESSES

So far there are two types of scalar field theories where the UV/IR mixing problem could be solved:

- the Grosse-Wulkenhaar model (2003), where the $\phi^4$ theory was supplemented by a (translation-invariance breaking) oscillator term ($\approx \tilde{x}^2\phi^2$)

# RECENT SUCCESSES

So far there are two types of scalar field theories where the UV/IR mixing problem could be solved:

- the Grosse-Wulkenhaar model (2003), where the $\phi^4$ theory was supplemented by a (translation-invariance breaking) oscillator term ($\approx \tilde{x}^2\phi^2$)
- and recently a translation-invariant model by Gurau, Magnen, Rivasseau and Tanasa (2008), where the oscillator term is replaced by a $\frac{1}{\tilde{p}^2}$ term.

# RECENT SUCCESSES

So far there are two types of scalar field theories where the UV/IR mixing problem could be solved:

- the Grosse-Wulkenhaar model (2003), where the $\phi^4$ theory was supplemented by a (translation-invariance breaking) oscillator term ($\approx \tilde{x}^2\phi^2$)
- and recently a translation-invariant model by Gurau, Magnen, Rivasseau and Tanasa (2008), where the oscillator term is replaced by a $\frac{1}{\tilde{p}^2}$ term.

$\Rightarrow$ *Both models could be proved to be renormalizable to all orders.*

# IN GAUGE THEORIES . . .

. . . matters are more complicated. Recent ansatzes:

- Slavnov model (2003): relies on a constraint → reduces degrees of freedom

# IN GAUGE THEORIES . . .

. . . matters are more complicated. Recent ansatzes:

- Slavnov model (2003): relies on a constraint → reduces degrees of freedom
- models involving oscillator terms in analogy to the scalar model: break translation invariance
  (de Goursac et. al., Grosse+Wohlgenannt, D.N.B. et. al., 2007)

# IN GAUGE THEORIES ...

...matters are more complicated. Recent ansatzes:

- Slavnov model (2003): relies on a constraint $\rightarrow$ reduces degrees of freedom
- models involving oscillator terms in analogy to the scalar model: break translation invariance
  (de Goursac et. al., Grosse+Wohlgenannt, D.N.B. et. al., 2007)
- proofs of renormalizability still missing for gauge theories

# IN GAUGE THEORIES . . .

. . . matters are more complicated. Recent ansatzes:

- Slavnov model (2003): relies on a constraint → reduces degrees of freedom
- models involving oscillator terms in analogy to the scalar model: break translation invariance
  (de Goursac et. al., Grosse+Wohlgenannt, D.N.B. et. al., 2007)
- proofs of renormalizability still missing for gauge theories

Inspired by the scalar model of Gurau et. al., whose propagator has “damping” behaviour for vanishing momentum:

$$G^{\phi\phi}(k) = \frac{1}{k^2 + m^2 + \frac{a}{\theta^2 k^2}} \approx \frac{\theta^2 k^2}{a} \quad \text{for } k \to 0\,,$$

a new gauge field model is put forward:

$$\int d^4x \phi(x) \frac{1}{\theta^2 \Box} \phi(x) \quad \Rightarrow \quad \int d^4x \frac{1}{4} F^{\mu\nu} \star \frac{1}{D^2 \widetilde{D}^2} \star F_{\mu\nu} \, ,$$

where

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - \mathrm{i}g \left[A_\mu \stackrel{\star}{,} A_\nu\right] \, ,$$
$$\widetilde{D}^2 = \widetilde{D}^\mu \star \widetilde{D}_\mu \, , \qquad \text{with} \;\; \widetilde{D}_\mu = \theta_{\mu\nu} D^\nu \, ,$$

# A NEW GAUGE FIELD MODEL



$$\int d^4x \phi(x) \frac{1}{\theta^2 \Box} \phi(x) \quad \Rightarrow \quad \int d^4x \frac{1}{4} F^{\mu\nu} \star \frac{1}{D^2 \widetilde{D}^2} \star F_{\mu\nu} \,,$$

where

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - \mathrm{i}g \left[A_\mu \stackrel{\star}{,} A_\nu\right] \,,$$
$$\widetilde{D}^2 = \widetilde{D}^\mu \star \widetilde{D}_\mu \,, \qquad \text{with } \widetilde{D}_\mu = \theta_{\mu\nu} D^\nu \,,$$

Expression $\frac{1}{D^2} F \equiv Y$ transforms covariantly ($sY = \mathrm{i}g \left[c \stackrel{\star}{,} Y\right]$) and is to be understood as formal power series in the gauge field $A_\mu$:

$$F = D^2 \star \frac{1}{D^2} \star F = D^2 Y =$$
$$= \Box Y - \mathrm{i}g \partial^\mu \left[A_\mu \stackrel{\star}{,} Y\right] - \mathrm{i}g \left[A^\mu \stackrel{\star}{,} \partial_\mu Y\right] + (\mathrm{i}g)^2 \left[A^\mu \stackrel{\star}{,} \left[A_\mu \stackrel{\star}{,} Y\right]\right] \,.$$

# COMPLETE ACTION INCLUDING GAUGE FIXING

- recursion formula leads to

$$Y^0 = \frac{1}{\Box} F \,,$$

$$Y^1 = \frac{1}{\Box} F + \frac{\mathrm{i}g}{\Box} \left\{ \partial^\mu \left[ A_\mu \stackrel{\star}{,} Y^0 \right] + \left[ A^\mu \stackrel{\star}{,} \partial_\mu Y^0 \right] - \mathrm{i}g \left[ A^\mu \stackrel{\star}{,} \left[ A_\mu \stackrel{\star}{,} Y^0 \right] \right] \right\}$$

and so on.

# Complete action including gauge fixing

- recursion formula leads to

$$Y^0 = \frac{1}{\Box} F \,,$$

$$Y^1 = \frac{1}{\Box} F + \frac{\mathrm{i}g}{\Box} \left\{ \partial^\mu \left[A_\mu \overset{\star}{,} Y^0\right] + \left[A^\mu \overset{\star}{,} \partial_\mu Y^0\right] - \mathrm{i}g \left[A^\mu \overset{\star}{,} \left[A_\mu \overset{\star}{,} Y^0\right]\right] \right\}$$

- New action (where stars have been suppressed):

$$\Gamma^{(0)} = S_{\mathsf{inv}} + S_{\mathsf{gf}} \,,$$

$$S_{\mathsf{inv}} = \int d^4x \left[ \frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \frac{1}{4} F^{\mu\nu} \frac{1}{D^2 \widetilde{D}^2} F_{\mu\nu} \right] \,,$$

$$S_{\mathsf{gf}} = s \int d^4x \, \bar{c} \left[ \left( 1 + \frac{1}{\Box \widetilde{\Box}} \right) \partial^\mu A_\mu - \frac{\alpha}{2} B \right]$$

$$= \int d^4x \left[ B \star \left( 1 + \frac{1}{\Box \widetilde{\Box}} \right) \partial^\mu A_\mu - \frac{\alpha}{2} B^2 - \bar{c} \left( 1 + \frac{1}{\Box \widetilde{\Box}} \right) \partial^\mu D_\mu c \right] .$$

# BRST symmetry and propagators

- this action is invariant under the BRST transformations

$$
\begin{aligned}
&sA_\mu = D_\mu c \equiv \partial_\mu c - \mathrm{i}g\left[A_\mu \stackrel{\star}{,} c\right], && s\bar{c} = B, \\
&sc = \mathrm{i}g c \star c, && sB = 0, \\
&s^2\varphi = 0 \qquad \text{for } \varphi \in \{A_\mu, c, \bar{c}, B\} \ .
\end{aligned}
$$

# BRST symmetry and propagators



- this action is invariant under the BRST transformations

$$
\begin{aligned}
&sA_\mu = D_\mu c \equiv \partial_\mu c - \mathrm{i}g\left[A_\mu \stackrel{\star}{,} c\right], && s\bar{c} = B, \\
&sc = \mathrm{i}gc \star c, && sB = 0, \\
&s^2\varphi = 0 \qquad \text{for } \varphi \in \{A_\mu, c, \bar{c}, B\}\ .
\end{aligned}
$$

- the bilinear action leads to the improved propagators

$$
G^A_{\mu\nu}(k) = \frac{1}{k^2 + \frac{1}{\tilde{k}^2}}\left(-\delta_{\mu\nu} + \frac{k_\mu k_\nu}{k^2} - \alpha\frac{k_\mu k_\nu}{k^2 + \frac{1}{\tilde{k}^2}}\right),
$$

$$
G^{\bar{c}c}(k) = \frac{1}{k^2 + \frac{1}{\tilde{k}^2}} \qquad \text{with} \quad \tilde{k}^\mu = \theta^{\mu\nu}k_\nu\,.
$$

# LOOP INTEGRALS

simplest non-planar one-loop integral:

$$\int d^4k \frac{e^{\pm \mathrm{i} k\tilde{p}}}{k^2 + \frac{1}{\tilde{k}^2}} = \frac{1}{2} \int d^4k \left[ \frac{e^{\pm \mathrm{i} k\tilde{p}}}{\left(k^2 + \frac{\mathrm{i}}{\theta}\right)} + \frac{e^{\pm \mathrm{i} k\tilde{p}}}{\left(k^2 - \frac{\mathrm{i}}{\theta}\right)} \right] \approx \frac{1}{\tilde{p}^2} \, .$$

for $\tilde{p}^2 \to 0$.

# LOOP INTEGRALS

simplest non-planar one-loop integral:

$$\int d^4k \frac{e^{\pm ik\tilde{p}}}{k^2 + \frac{1}{\tilde{k}^2}} = \frac{1}{2} \int d^4k \left[ \frac{e^{\pm ik\tilde{p}}}{\left(k^2 + \frac{i}{\theta}\right)} + \frac{e^{\pm ik\tilde{p}}}{\left(k^2 - \frac{i}{\theta}\right)} \right] \approx \frac{1}{\tilde{p}^2} \, .$$

for $\tilde{p}^2 \to 0$.

- In the sum of 1-loop graphs due to gauge symmetry:

$$\approx \frac{\tilde{p}_\mu \tilde{p}_\nu}{(\tilde{p}^2)^2}$$

# LOOP INTEGRALS

simplest non-planar one-loop integral:

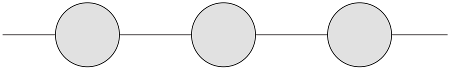
$$\int d^4k \frac{e^{\pm ik\tilde{p}}}{k^2 + \frac{1}{\tilde{k}^2}} = \frac{1}{2}\int d^4k \left[\frac{e^{\pm ik\tilde{p}}}{\left(k^2 + \frac{\mathrm{i}}{\theta}\right)} + \frac{e^{\pm ik\tilde{p}}}{\left(k^2 - \frac{\mathrm{i}}{\theta}\right)}\right] \approx \frac{1}{\tilde{p}^2}\,.$$

for $\tilde{p}^2 \to 0$.

- In the sum of 1-loop graphs due to gauge symmetry:

$$\approx \frac{\tilde{p}_\mu \tilde{p}_\nu}{(\tilde{p}^2)^2}$$

- IR damping due to propagators



# More convenient formulation

$$S_{\text{new}} = \int d^4x \left[ B^{\mu\nu} \star F_{\mu\nu} - B^{\mu\nu} \star \widetilde{D}^2 D^2 \star B_{\mu\nu} \right]$$

is gauge invariant if new field $B_{\mu\nu}$ transforms covariantly, i.e.

$$sB_{\mu\nu} = \mathrm{i}g \left[ c \stackrel{\star}{,} B_{\mu\nu} \right]$$

and has only a finite number of new vertices.

# More convenient formulation

$$S_{\mathsf{new}} = \int d^4x \left[ B^{\mu\nu} \star F_{\mu\nu} - B^{\mu\nu} \star \widetilde{D}^2 D^2 \star B_{\mu\nu} \right]$$

is gauge invariant if new field $B_{\mu\nu}$ transforms covariantly, i.e.

$$sB_{\mu\nu} = \mathrm{i}g \left[ c \stackrel{\star}{,} B_{\mu\nu} \right]$$

and has only a finite number of new vertices.
Re-inserting the e.o.m.

$$\frac{\delta S_{\mathsf{new}}}{\delta B^{\rho\sigma}} = F_{\rho\sigma} - 2\widetilde{D}^2 D^2 \star B_{\rho\sigma} = 0$$

leads again to

$$\int d^4x \frac{1}{4} F^{\mu\nu} \star \frac{1}{D^2 \widetilde{D}^2} \star F_{\mu\nu} \,,$$

# additional Feynman rules

- two additional propagators:

$$G^{AB}_{\rho,\sigma\tau}(k) = \frac{\mathrm{i}\left(k_\sigma\delta_{\rho\tau} - k_\tau\delta_{\rho\sigma}\right)}{2k^2\tilde{k}^2\left(k^2 + \frac{1}{\tilde{k}^2}\right)},$$

$$G^{BB}_{\rho\sigma,\tau\epsilon}(k) = \frac{1}{4k^2\tilde{k}^2}\Bigg[\delta_{\rho\tau}\delta_{\sigma\epsilon} - \delta_{\rho\epsilon}\delta_{\sigma\tau} + \\ + \frac{k_\sigma k_\tau\delta_{\rho\epsilon} + k_\rho k_\epsilon\delta_{\sigma\tau} - k_\sigma k_\epsilon\delta_{\rho\tau} - k_\rho k_\tau\delta_{\sigma\epsilon}}{k^2\tilde{k}^2\left(k^2 + \frac{1}{\tilde{k}^2}\right)}\Bigg].$$

# additional Feynman rules

- two additional propagators:

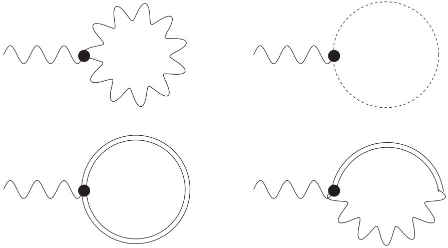
$$G^{AB}_{\rho,\sigma\tau}(k) = \frac{\mathrm{i}\left(k_\sigma \delta_{\rho\tau} - k_\tau \delta_{\rho\sigma}\right)}{2k^2\tilde{k}^2\left(k^2 + \frac{1}{\tilde{k}^2}\right)},$$

$$G^{BB}_{\rho\sigma,\tau\epsilon}(k) = \frac{1}{4k^2\tilde{k}^2}\Big[\delta_{\rho\tau}\delta_{\sigma\epsilon} - \delta_{\rho\epsilon}\delta_{\sigma\tau} + \\ + \frac{k_\sigma k_\tau \delta_{\rho\epsilon} + k_\rho k_\epsilon \delta_{\sigma\tau} - k_\sigma k_\epsilon \delta_{\rho\tau} - k_\rho k_\tau \delta_{\sigma\epsilon}}{k^2\tilde{k}^2\left(k^2 + \frac{1}{\tilde{k}^2}\right)}\Big].$$

- and 5 new vertices, namely a $BAA$-vertex, a $BBA$-vertex, a $2B2A$-vertex, a $2B3A$-vertex and a $2B4A$-vertex.

# VANISHING TADPOLE GRAPHS

There are only 4 possible one-loop tadpole graphs with external gauge boson lines in this model:



From the Feynman rules one sees that all 4 come with a factor

$$\delta^4(p+k-k)\sin\left(\frac{k\theta p}{2}\right) \to 0\,,$$

*Hence, all four graphs vanish.*

# Conclusion and Outlook



- the model seems to be a promising candidate for a renormalizable non-commutative gauge model

# Conclusion and Outlook

- the model seems to be a promising candidate for a renormalizable non-commutative gauge model
- gauge field and ghost propagators damp IR divergences

# Conclusion and Outlook

- the model seems to be a promising candidate for a renormalizable non-commutative gauge model
- gauge field and ghost propagators damp IR divergences
- it is translation-invariant (tadpoles vanish)

# Conclusion and Outlook



- the model seems to be a promising candidate for a renormalizable non-commutative gauge model
- gauge field and ghost propagators damp IR divergences
- it is translation-invariant (tadpoles vanish)
- one-loop calculations are in progress

# References




- D. N. Blaschke, F. Gieres, E. Kronberger, M. Schweda and M. Wohlgenannt, to appear in J. Phys. A, [arXiv:0804.1914].
- R. Gurau, J. Magnen, V. Rivasseau and A. Tanasa, [arXiv:0802.0791].
- H. Grosse and F. Vignes-Tourneret, [arXiv:0803.1035].
- F. Aigner, M. Hillbrand, J. Knapp, G. Milovanovic, V. Putz, R. Schoefbeck and M. Schweda, *Czech. J. Phys.* **54** (2004) 711–719, [arXiv:hep-th/0302038].


*Thank you for your attention!*